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NAVORD REPORT

NOTS TP 2134   NAVORD REPORT 6428

HYDRODYNAMIC DESIGN CRITERIA FOR ADEQUATE TORPEDO STABILITY AND RESPONSE

By
Calvin W. Sweat

DEPARTMENT OF THE NAVY
NAVAL ORDNANCE TEST STATION

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U N C L A S S I F I E D

U. S. NAVAL ORDNANCE TEST STATION

W. W. Hollister, Capt., USN
Commander

Wm. B. McLean, Ph.D.
Technical Director

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Copy № 5

China Lake, California
29 October 1958

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NOTS Technical Publication 2134
NAVORD Report 6428

Published by . . . Underwater Ordnance Department
Manuscript 804/MS-106
Collation . . . . . Cover, 34 leaves, abstract cards
First printing . . . . 130 numbered copies
Security classification . . . . UNCLASSIFIED
Abstract alone . . . . UNCLASSIFIED
Title alone . . . . UNCLASSIFIED

FOREWORD

The work described in this report is part of the continuing study of hydrodynamics at this Station. The report analyzes the effect of body shape, tail and rudder size, and other physical parameters on the stability and response of torpedoes. The results of this analysis will be useful in preliminary design and control studies.

The work was performed between June 1957 and June 1958 under Bureau of Ordnance Task Assignment NO-404-664/41001/01060. The report was reviewed for technical accuracy by H. T. Yerby and L. A. Lopes of this Station.

D. J. WILCOX, Head
Underwater Ordnance
Department

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WM. B. McLEAN
Technical Director

ABSTRACT

The stability and response of torpedoes are affected by variations in body shape, weight, volume, center of gravity, tail size, rudder size, and velocity. Two hydrodynamic design criteria can be developed which provide adequate stability and good response. All torpedoes of conventional shape, designed to the criterion L*, a dimensionless tail-lift coefficient, respond to stepwise rudder deflections in an almost identical manner, provided that the steps result in equal changes in the steady-state turn rate. To obtain such almost identical response in the special case of linear proportional control, a second criterion K*, characteristic for control gain, is required. These two criteria make it possible to match the response of widely different torpedo bodies both to constant-rudder and to linear proportional control.

CONTENTS

NOMENCLATURE

A	Maximum cross-sectional area, ft
a, b, c, d e, f, g	Constants defined in Appendix A
c_p	Prismatic coefficient, $c_p = \dfrac{\forall}{A\ell}$
D'_o	$\dfrac{\text{Drag}}{(1/2)\rho AV^2}$
J_z	$(1/2)\rho A\ell^3 J'_z$, moment of inertia about z axis, slug ft^2
K	A constant defined in the control equation
K^*	A particular value of K
L^*	A particular value of L'_{β_T} defined on page 3
L'_{β_T}	$\dfrac{\text{Tail-lift derivative}}{(1/2)\rho AV^2}$
ℓ	Total length, ft
ℓ_c	Cylindrical-section length, ft
ℓ_n	Nose length, ft
ℓ_p	$\ell_n + \ell_t$, ft
ℓ_t	Tail length
m_1, ω, ζ	m_1 and $-\omega\zeta \pm \omega\sqrt{1 - \zeta^2}\, i$ are the roots of the cubic equation associated with the yaw equation with linear proportional control

m^*_1 A particular value of m_1

m_L Apparent longitudinal mass (Ref. 2, pp. 22 - 23)
$\frac{1}{2}\rho A\ell m'_L$

m_T Apparent transverse mass
$\frac{1}{2}\rho A\ell m'_T$

N_β Moment derivative with respect to β, ft lb

N_{β_B} $(1/2)\rho A\ell V^2 N'_{\beta_B}$ = body-moment derivative, ft lb

N'_{δ_r} $\dfrac{\text{Rudder-force derivative}}{(1/2)\rho A\ell V^2} = \dfrac{x_\delta}{\ell} Y'_{\delta_r}$

N_r Damping-moment derivative, ft lb

N_{r_B} $(1/2)\rho A\ell^2 V N'_{r_B}$ = body-damping-moment derivative, ft lb

r z-component of angular velocity

t Time

V Speed, ft/sec

$\forall$ Volume of torpedo, ft^3

x, y, z Right-hand coordinate system, fixed in torpedo body, originating on torpedo axis at the point nearest the center of gravity; x-axis along torpedo axis positive forward, z positive downward

x_o, y_o, z_o Fixed, right-hand coordinate system with z_o positive downward

x_δ Distance between origin of moving coordinate system and center of rudder lift, ft

x_ρ Distance between origin of moving coordinate system and center of tail lift, ft

Y_β Force derivative with respect to β, lb

Y_{β_B} $(1/2)\rho AV^2 Y'_{\beta_B}$ = body-force derivative with respect to β, lb

Y'_{δ_r} $\dfrac{\text{Rudder-force derivative}}{(1/2)\rho AV^2}$

Y_r Damping-force derivative, lb sec

Y_{r_B} $(1/2)\rho A\ell VY'_{r_B}$ = body-damping-force derivative, lb sec

β Angle of attack in yaw = angle between the velocity vector and its projection on the x-z plane (measured in direction from the projection toward the y axis)

δ_r Rudder-deflection angle, positive in direction from -x axis to -y axis, rad

ζ Damping factor

η ℓ_n/ℓ_p

θ Pitch angle: angle between projection of x axis on x_o - y_o plane and x axis (measured from x_o - y_o plane in direction of $-z_o$ axis), rad

ρ Density of fluid, slugs/ft^3; $\rho \simeq 2$ slugs/ft^3 for salt water

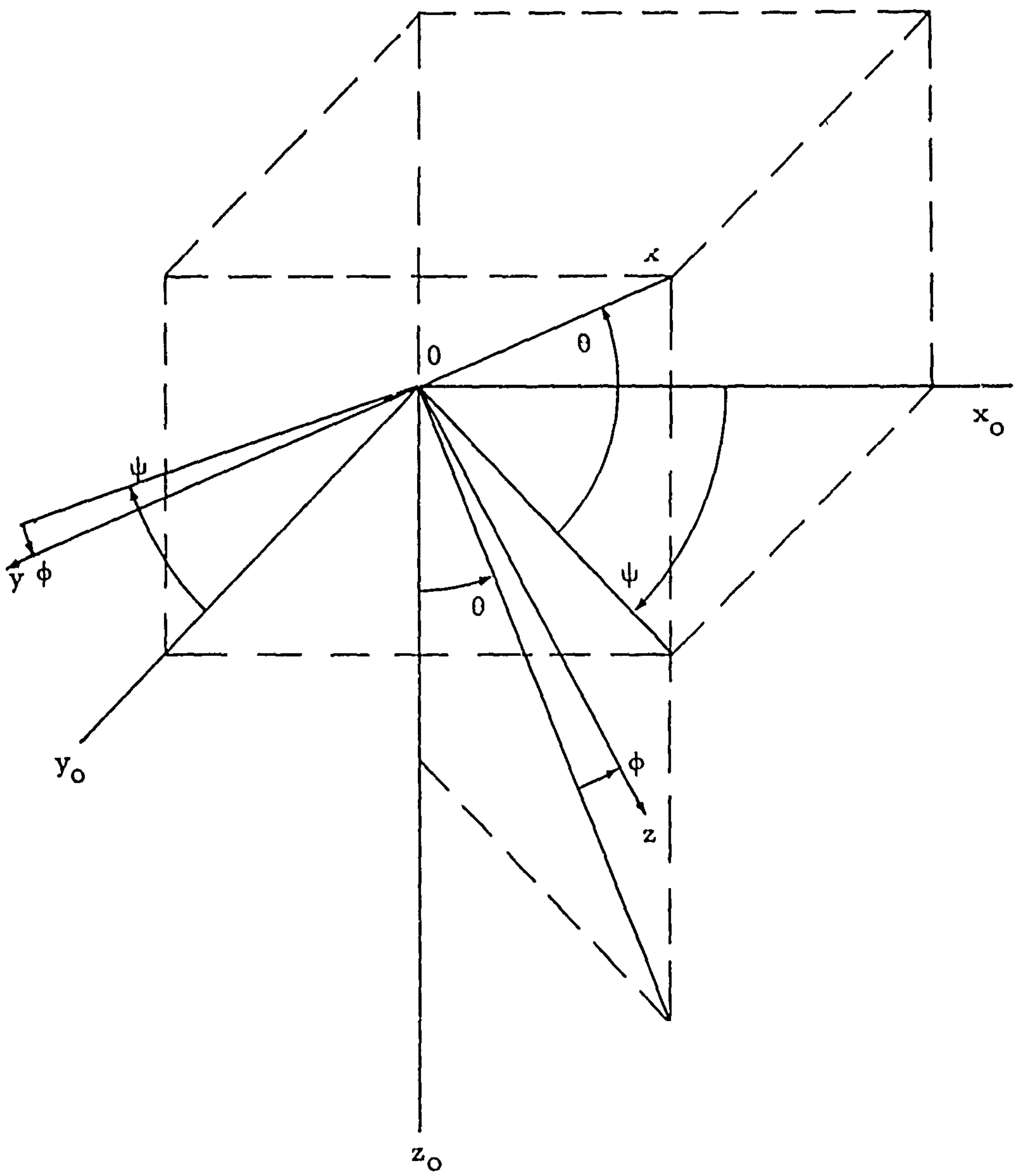
ϕ Roll angle: angle between plane containing x and z_o axes and -z axis (measured from the plane to the -z axis in direction from -z to y), rad

ψ Yaw angle: angle between x_o axis and the projection of x axis on x_o - y_o plane (measured from x_o in direction of y_o), rad

$\dot{\psi}$ $\dfrac{d\psi}{dt}$

$\dot{\psi}_{ss}$ Steady-state turn rate

ω Natural frequency



Coordinated Systems Related

INTRODUCTION

The ability of a torpedo to pursue a target successfully requires that an error in heading be corrected quickly, without objectionable overshoot or oscillation; these corrections should be made by controls which are not excessively large or powerful. This report presents hydrodynamic design criteria for a wide variety of body shapes which give adequate stability and have good response characteristics. Moreover, the response of all torpedoes designed to the criteria will be almost identical, so that the question of matching the response of widely different bodies is at least partially answered.

A group of ten torpedo bodies, with the most important body parameters varied systematically to include most practical configurations, was studied to show the effect of body shape on stability and response. The investigation was limited to the yaw plane, but the response in the pitch plane is similar, except for the effects of weight and buoyancy.

CONTROL SYSTEM

In order to develop the desired design criteria, a type of control system has to be selected. One of the simplest, insofar as instrumentation and circuitry are concerned, is linear proportional control. If ψ is the yaw error, the rudder deflection δ_r is given by $\delta_r = C\psi$, where C is a constant called the control gain (see Nomenclature). Analysis of a linear proportional control system may lead to excessive component sizes and suggest a non-linear system or the incorporation of derivative control.

However, because derivative control or feedback from control surfaces introduces added instrumentation, networks, and amplifiers, the use of linear proportional control was assumed. This assumption may not be too restrictive, since a torpedo having good control characteristics with this system may be expected to respond adequately to other types of control. For the torpedo bodies considered in this report, linear proportional control results in reasonable hydrodynamic design requirements.

DESIGN CRITERIA FOR ADEQUATE STABILITY AND GOOD RESPONSE

According to Appendix A, if the control equation is

$$\delta_r = \frac{K}{\frac{x_\delta}{\ell} Y'_{\delta_r}} \psi \tag{1}$$

then the yaw-error equation is

$$\dddot{\psi} + (a + bL'_{\beta_T})\ddot{\psi} + (c + dL'_{\beta_T} + eK)\dot{\psi} + (Kf)\psi = 0 \tag{2}$$

where a, b, c, d, e, and f are constants for a particular torpedo, as defined in Appendix A. Equation 2 has the general solution

$$\psi = A \exp(m_1 t) + [B \cos(\omega\sqrt{1 - \zeta^2}\, t) + C \sin(\omega\sqrt{1 - \zeta^2}\, t)] \exp(-\omega\zeta t) \tag{3}$$

where m_1 and $-\omega\zeta \pm \omega\sqrt{1 - \zeta^2}\, i$ are the roots of the auxiliary equation associated with Eq. 2. Since ψ is to approach zero, m_1 must be negative, for stability. The quantities ω and ζ are called, respectively, the natural frequency and the damping factor. If ω and ζ are specified, then (from Appendix B)

(4) $$L'_{\beta_T}(\omega, \zeta) = \frac{2e\zeta\omega^3 + (f - 4f\zeta^2 - ae)\omega^2 + 2af\zeta\omega - cf}{be\omega^2 - 2bf\zeta\omega + df}$$

(5) $$K(\omega, \zeta) = \frac{b\omega^4 - 2d\zeta\omega^3 + (da - bc)\omega^2}{be\omega^2 - 2bf\zeta\omega + df}$$

and

(6) $$m_1(\omega, \zeta) = \frac{-fK(\omega, \zeta)}{\omega^2}$$

where

$$L'_{\beta_T} = \frac{\text{tail lift}}{(1/2)\rho A V^2}$$

where ρ = density of water. Appendix D gives the derivation of L'_{β_T} as a function of fin size. The values $\omega = 2\pi$ and $\zeta = 0.707$ are considered quite satisfactory for frequency and damping.[1] The corresponding dimensionless tail-lift coefficient, $L^* = L'_{\beta_T}(2\pi, 0.707)$, and the constant K from the control equation, $K^* = K(2\pi, 0.707)$, will then be taken as the design criteria. If also $m_1^* = m_1(2\pi, 0.707)$ then, from Eq. 4, 5, and 6

(7) $$L^* = \frac{f(8.886a - c - 39.48) + e(350.7 - 39.48a)}{f(d - 8.886b) + 39.48be}$$

(8) $$K^* = \frac{b(1558 - 39.48c) + d(39.48a - 350.7)}{f(d - 8.886b) + 39.48be}$$

and

(9) $$m_1^* = \frac{-fK^*}{39.48}$$

[1] These values were suggested by L. A. Lopes and D. E. Elliott on the basis of past experience.

For these values Eq. 3 becomes

$$(10) \qquad \psi = A \exp\left(\frac{-fK^*}{39.48}t\right) + (B \cos 4.443t + C \sin 4.443t)\exp(-4.443t)$$

If, as is often the case, the rudder deflection is limited to the range $(-\delta r_o, \delta r_o)$ then, from Eq. 1, the range in ψ (referred to as the "band of proportional control") for which Eq. 9 applies is

$$(11) \qquad \left(\frac{\frac{-x_\delta}{\ell} Y'_{\delta_r} \delta_{r_o}}{K^*}, \quad \frac{\frac{x_\delta}{\ell} Y'_{\delta_r} \delta_{r_o}}{K^*}\right)$$

Thus the criteria L* and K* lead to good control characteristics for small yaw errors. If, however, the magnitude of the yaw error ψ is greater than

$$\frac{\frac{x_\delta}{\ell} Y'_{\delta_r} \delta_{r_o}}{K^*}$$

the rudder will be "hard over" and the resulting yaw response will be of importance. The yaw equation is then

$$(12) \qquad \dddot{\psi} + (a + bL^*)\ddot{\psi} + (c + dL^*)\dot{\psi} = \left(\begin{array}{ll} -f\frac{x_\delta}{\ell} Y'_{\delta_r} \delta_{r_o}, & \text{if } \psi > \dfrac{\frac{x_\delta}{\ell} Y'_{\delta_r} \delta_{r_o}}{K^*} \\ f\frac{x_\delta}{\ell} Y'_{\delta_r} \delta_{r_o}, & \text{if } \psi < \dfrac{\frac{-x_\delta}{\ell} Y'_{\delta_r} \delta_{r_o}}{K^*} \end{array}\right)$$

The steady-state solution of this equation is

$$\dot{\psi}_{ss} = \mp \frac{f \frac{x_\delta}{\ell} Y'_{\delta_r} \delta_{r_o}}{c + dL^*} \tag{13}$$

Here the main questions are whether or not, for a given torpedo body and a given steady-state turn rate, the design criterion L^* will lead to a reasonably quick adjustment to the steady-state turn rate, and how the response times will differ for two different bodies. It will be seen later that the answers to both questions are favorable; these criteria give good results for large errors (hard-over rudder) as well as small ones (proportional control).

PARAMETERS OF SAMPLE TORPEDOES

A group of ten torpedo bodies is defined in Table 1 and shown in Fig. 1. The nose of each torpedo is assumed to be elliptical and the tail parabolic, according to equations given in an earlier report.[2] If a nose has a flat portion, its effect is assumed to be too small to be significant. The three parameters $\eta = \ell_n/(\ell_n + \ell_t)$, $\ell_p/d = (\ell_n + \ell_t)/d$, and ℓ_c/ℓ, defined in Table 1, are then sufficient to describe the torpedo shape. These parameters are varied independently through ranges which include most practical torpedo configurations, so that conclusions concerning these bodies will be generally applicable. Additional assumptions are:

1. Speed V = 40 knots.
2. Volume $\forall$ = 5 ft^3.
3. Specific gravity 1.2, weight uniformly distributed.

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[2] U. S. Naval Ordnance Test Station. A Fast Method for Finding the Physical Characteristics of a Torpedo, by D. Argue. Pasadena, Calif., 22 August 1956. (G. C. Memorandum 685.)

TABLE 1. Physical Parameters of Sample Torpedoes

Body	$\eta = \dfrac{\ell_n}{\ell_n + \ell_t}$	$\dfrac{\ell_p}{d} = \dfrac{\ell_n + \ell_t}{d}$	$\dfrac{\ell_c}{\ell}$	$\dfrac{\ell}{d}$	ℓ
1	0.1	3	0	3	4.716
1.1	0.1	3	0.6	7.5	7.590
2	0.1	5	0	5	6.630
2.1	0.1	5	0.6	12.5	10.675
3	0.4	3	0	3	4.605
3.1	0.4	3	0.5	6	6.612
4	0.4	4	0	4	5.580
4.1	0.4	4	0.7	13.3	10.893
5	0.4	5	0	5	6.475
5.1	0.4	5	0.5	10	9.290

d = diameter

ℓ = total length

ℓ_c = length of cylindrical section

ℓ_n = length of nose section

ℓ_t = length of tail section

$\ell_p = \ell_n + \ell_t$

V = 40 knots

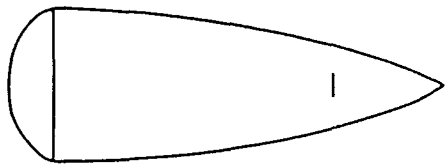
$\mathcal{V} = 5$ ft^3

4. The dimensionless drag coefficient, based on maximum cross-sectional area, is 0.1 (this is not a critical quantity).
5. $x_\delta = 1.05\, x_\rho$; x_ρ is the distance, in feet, from the CG to the center of tail lift ($x_\rho < 0$); x_δ is the distance from the CG to the rudder center of lift.
6. The tail lift acts at a distance $\ell - 0.293\ell_t - 0.167$ feet from the nose. This is approximately the point where the body diameter is one half the maximum body diameter.
7. When the rudders are deflected, there is a certain time lag before the full lift develops. According to unsteady-

airfoil theory, this time lag approximates the time required to travel three or four chord lengths or, for all cases considered here, about 0.01 - 0.02 second. Therefore this effect was neglected.

The physical characteristics and dimensionless hydrodynamic body coefficients, necessary for the determination of the quantities a, b, c, d, e, and f, are found for each body from Ref. 1 and 2 and the report referred to in footnote 2. Then L^*, K^*, and m_1^* are computed from Eq. 7, 8, and 9. Table 2 gives these values, the presently used stability criterion G (Ref. 2) and $-Y'_{\delta_r}/(\dot{\psi}_{ss}/\delta_{r_o})$ from Eq. 13. Appendix C presents the computation of the criteria for one of the bodies (Body 1.1) in order to demonstrate the method by a practical example. It turns out that the smallest value of $-m_1^*$ is 11.1, so that in each case the effect of the root m_1^* can be neglected after a short period of time (see Appendix B). Other conclusions are: For a given nose and tail shape, both L^* and $-K^*$ increase by adding a cylindrical section. The addition of a cylindrical section increases the Y'_{δ_r} required for a given steady-state turn rate. The effect of the relative lengths of the nose and tail sections may be assessed by comparing Bodies 1 and 3 or 2 and 5. As the ratio ℓ_n/ℓ_t increases [holding $(\ell_n + \ell_t)/d$ constant], L^* increases while K^* and $Y'_{\delta_r}/(\dot{\psi}_{ss}/\delta_{r_o})$ remain essentially unchanged. Conversely, with a constant ℓ_n/ℓ_t, the effect of increasing $(\ell_n + \ell_t)/d$ is to increase L^*, K^*, and $Y'_{\delta_r}/(\dot{\psi}_{ss}/\delta_{r_o})$.

1

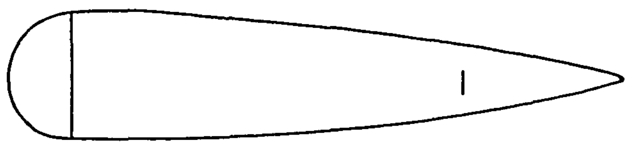


$\frac{CB}{\ell} = \frac{CG}{\ell} = 0.3427$, $c_p = 0.5464$, $Y'_{\beta_B} = 0.7$ $Y'_{r_B} = 0.065$ $m'_L = 0.5464$ (2 sp. gr. + 0.26)
$N'_{\beta_B} = 0.608$, $N'_{r_B} = -0.03$, $m'_T = 0.5464$ (2 sp. gr. + 1.6), $J'_z = 0.0275 + 0.0598$ sp. gr

1.1

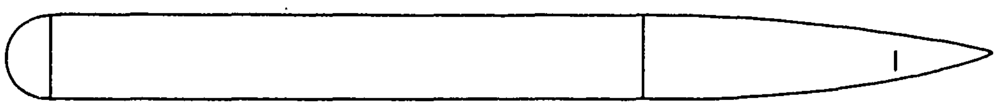
$\frac{CB}{\ell} = \frac{CG}{\ell} = 0.426$, $c_p = 0.8186$, $Y'_{\beta_B} = 0.54$ $Y'_{r_B} = 0.30$ $m'_L = 0.8186$ (2 sp. gr. + 0.05)
$N'_{\beta_B} = 1.1$, $N'_{r_B} = -0.15$, $m'_T = 0.8186$ (2 sp. gr. + 1.82), $J'_z = 0.0815 + 0.0988$ sp. gr.

2



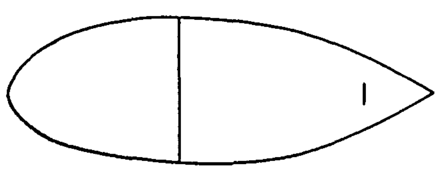
$\frac{CB}{\ell} = \frac{CG}{\ell} = 0.343$, $c_p = 0.5464$, $Y'_{\beta_B} = 0.73$ $Y'_{r_B} = 0.063$ $m'_L = 0.5464$ (2 sp. gr. + 0.11)
$N'_{\beta_B} = 0.608$, $N'_{r_B} = -0.03$, $m'_T = 0.5464$ (2 sp. gr. + 1.78), $J'_z = 0.0346 + 0.0494$ sp. gr

2.1



$\frac{CB}{\ell} = \frac{CG}{\ell} = 0.427$, $c_p = 0.6186$, $Y'_{\beta_B} = 0.75$ $Y'_{r_B} = 0.30$ $m'_L = 0.8186$ (2 sp. gr. + 0.02)
$N'_{\beta_B} = 1.1$, $N'_{r_B} = -0.15$, $m'_T = 0.8186$ (2 sp. gr. + 1.96), $J'_z = 0.0886 + 0.0973$ sp. gr

3



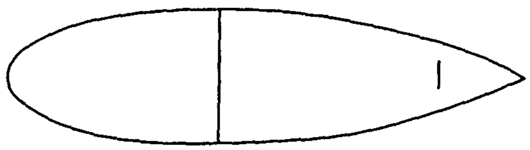
$\frac{CB}{\ell} = \frac{CG}{\ell} = 0.434$, $c_p = 0.5866$, $Y'_{\beta_B} = 0.66$ $Y'_{r_B} = 0.085$ $m'_L = 0.5866$ (2 sp. gr. + 0.26)
$N'_{\beta_B} = 0.77$, $N'_{r_B} = -0.04$, $m'_T = 0.5866$ (2 sp. gr. + 1.6), $J'_z = 0.0257 + 0.0558$ sp. gr

FIG. 1. Sample Torpedo Bodies.

3.1

$\frac{CB}{l} = \frac{CG}{l} = 0.465$, $c_p = 0.7933$, $Y'_{\beta_B} = 0.52$ $Y'_{r_B} = 0.26$ $m'_L = 0.7933$ (2 sp.gr + 0.08)
$N'_{\beta_B} = 1.1$, $N'_{r_B} = -0.13$, $m'_T = 0.7933$ (2 sp gr + 1.84), $J'_z = 0.0692 + 0.0911$ sp gr

4

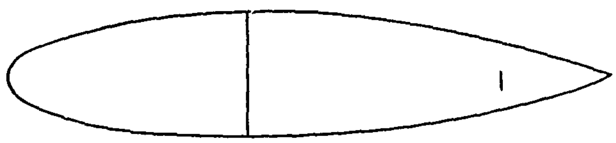


$\frac{CB}{l} = \frac{CG}{l} = 0.434$, $c_p = 0.5866$, $Y'_{\beta_B} = 0.66$ $Y'_{r_B} = 0.09$ $m'_L = 0.5866$ (2 sp gr + 0.16)
$N'_{\beta_B} = 0.81$, $N'_{r_B} = -0.043$, $m'_T = 0.5866$ (2 sp gr + 1.72), $J'_z = 0.0319 + 0.0523$ sp gr

4.1

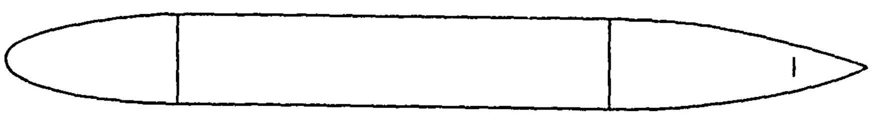
$\frac{CB}{l} = \frac{CG}{l} = 0.478$, $c_p = 0.8759$, $Y'_{\beta_B} = 0.74$ $Y'_{r_B} = 0.41$ $m'_L = 0.8759$ (2 sp gr + 0.02)
$N'_{\beta_B} = 1.28$, $N'_{r_B} = -0.20$, $m'_T = 0.8759$ (2 sp gr + 1.96), $J'_z = 0.1055 + 0.1147$ sp gr

5



$\frac{CB}{l} = \frac{CG}{l} = 0.434$, $c_p = 0.5866$, $Y'_{\beta_B} = 0.69$ $Y'_{r_B} = 0.085$ $m'_L = 0.5866$ (2 sp.gr + 0.11)
$N'_{\beta_B} = 0.81$, $N'_{r_B} = -0.04$, $m'_T = 0.5866$ (2 sp gr + 1.8), $J'_z = 0.0361 + 0.0515$ sp gr

5.1



$\frac{CB}{l} = \frac{CG}{l} = 0.465$, $c_p = 0.7933$, $Y'_{\beta_B} = 0.66$ $Y'_{r_B} = 0.27$ $m'_L = 0.7933$ (2 sp gr + 0.04)
$N'_{\beta_B} = 1.28$, $N'_{r_B} = -0.135$, $m'_T = 0.7933$ (2 sp.gr + 1.92), $J'_z = 0.0787 + 0.0894$ sp gr

FIG. 1. (Contd.)

TABLE 2. Comparison of Sample Bodies

Body	L^*	$-K^*$	$\dfrac{-Y'_{\delta_r}}{\dot{\psi}_{ss}/\delta_{r_o}}$	$-m_1^*$	G (Ref. 2)
1	1.30 (11,530)[1]	0.041	0.0211 (187.1)	31.8	0.665
1.1	1.84 (6,763)	0.270	0.0813 (298.8)	19.5	0.800
2	1.31 (8,267)	0.079	0.0352 (222.1)	22.1	0.758
2.1	1.81 (4,738)	0.428	0.1176 (307.8)	11.1	0.818
3	1.56 (13,192)	0.042	0.0216 (182.7)	41.1	0.618
3.1	1.93 (8,412)	0.189	0.0667 (290.7)	24.7	0.781
4	1.63 (11,384)	0.064	0.0310 (216.5)	34.1	0.683
4.1	2.06 (4,959)	0.513	0.1471 (354.1)	11.1	0.846
5	1.65 (9,931)	0.086	0.0394 (237.1)	27.8	0.731
5.1	2.04 (6,319)	0.332	0.1053 (326.2)	16.2	0.826

[1]Numbers in parentheses represent the dimensional coefficients, i.e., $(1/2)\rho AV^2L^*$ and $\dfrac{-(1/2)\rho AV^2 Y'_{\delta_r}}{\dot{\psi}_{ss}/\delta_{r_o}}$.

YAW-ERROR CORRECTION FOR SAMPLE TORPEDOES

Figure 2 shows the correction of a 20-degree yaw error for each of the ten bodies described in the preceding section, under the assumption that the rudder deflection is limited to ± 6 degrees and that the steady-state turn rate for this deflection is ∓ 20 deg/sec. The initial conditions are

$$\psi = 20 \text{ degrees}$$
$$\dot{\psi} = 0$$
$$\ddot{\psi} = 6 \frac{V^2}{\ell^2} \frac{N'_{\delta_r}}{J'_z} \text{ deg/sec}^2$$

at t = 0. The error is corrected in slightly more than one second, with very slight overshoot. That the design criteria L* and K* lead to torpedoes with quite similar responses is readily apparent since the curves essentially overlap.

The curves in Fig. 2 are plotted without regard for the time lag in the control, under the assumption that the rudder is instantaneously at the angle prescribed by the control Eq. 1. Practically, however, the rudder "lags" behind the desired angle. Therefore the control is more accurately described by $T_p\dot{\delta}_r + \delta_r = K^*/(x_\delta/\ell) Y'_{\delta_r}$, where T_p is the time lag. Figures 3 through 12 show, for $L'_{\beta_T} = L^*$ and $K = K^*$, the effects of a 0.05-second and 0.25-second time lag on each body. The 0.05-second lag produces no great effect on the response, resulting only in slightly more overshoot. A time lag of 0.25 second, however, results in relatively wide and prolonged oscillations. Therefore the time lag should be held to a minimum, on the order of 0.05 second or less.

Thus far it has been demonstrated that, independent of body shape, the criteria L* and K* result in torpedoes with very similar responses, under the assumptions stated in the preceding

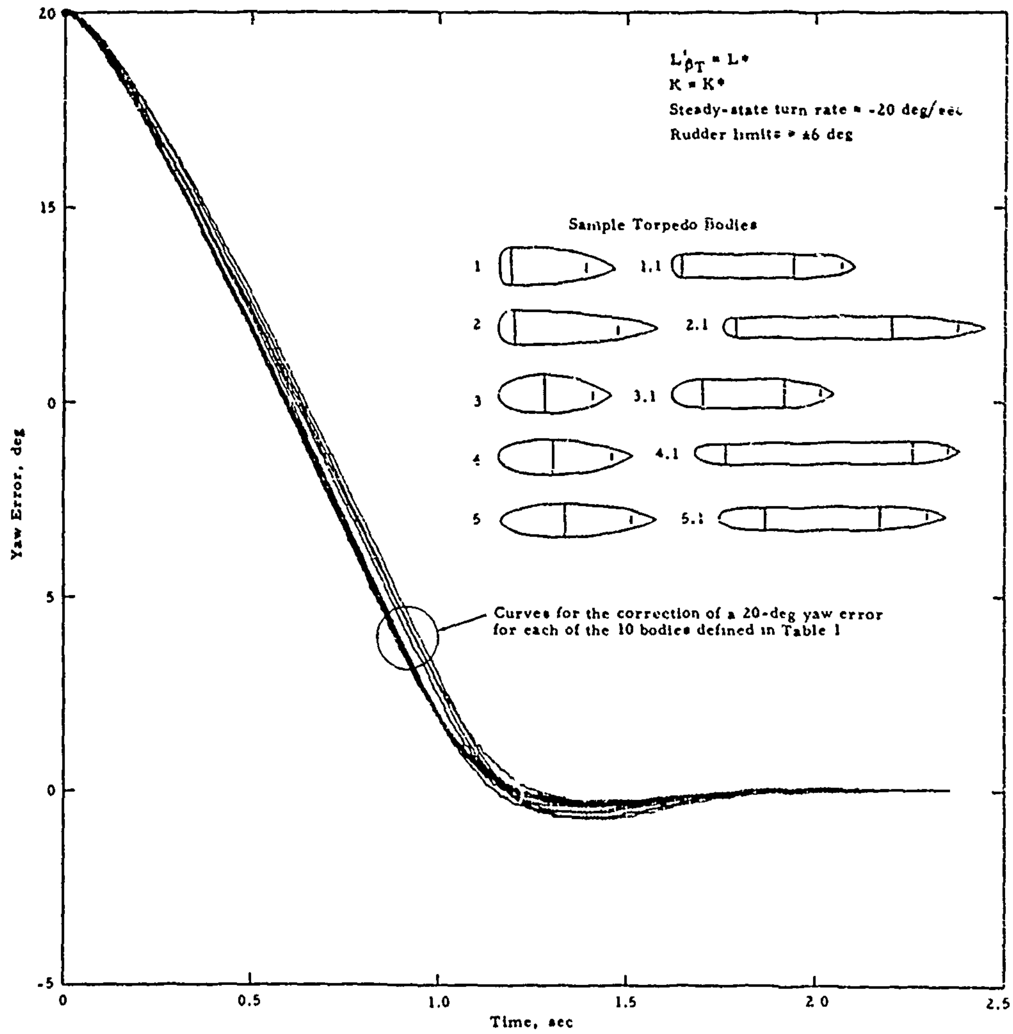


FIG. 2. Correction of Yaw Error for Each of the Sample Bodies.

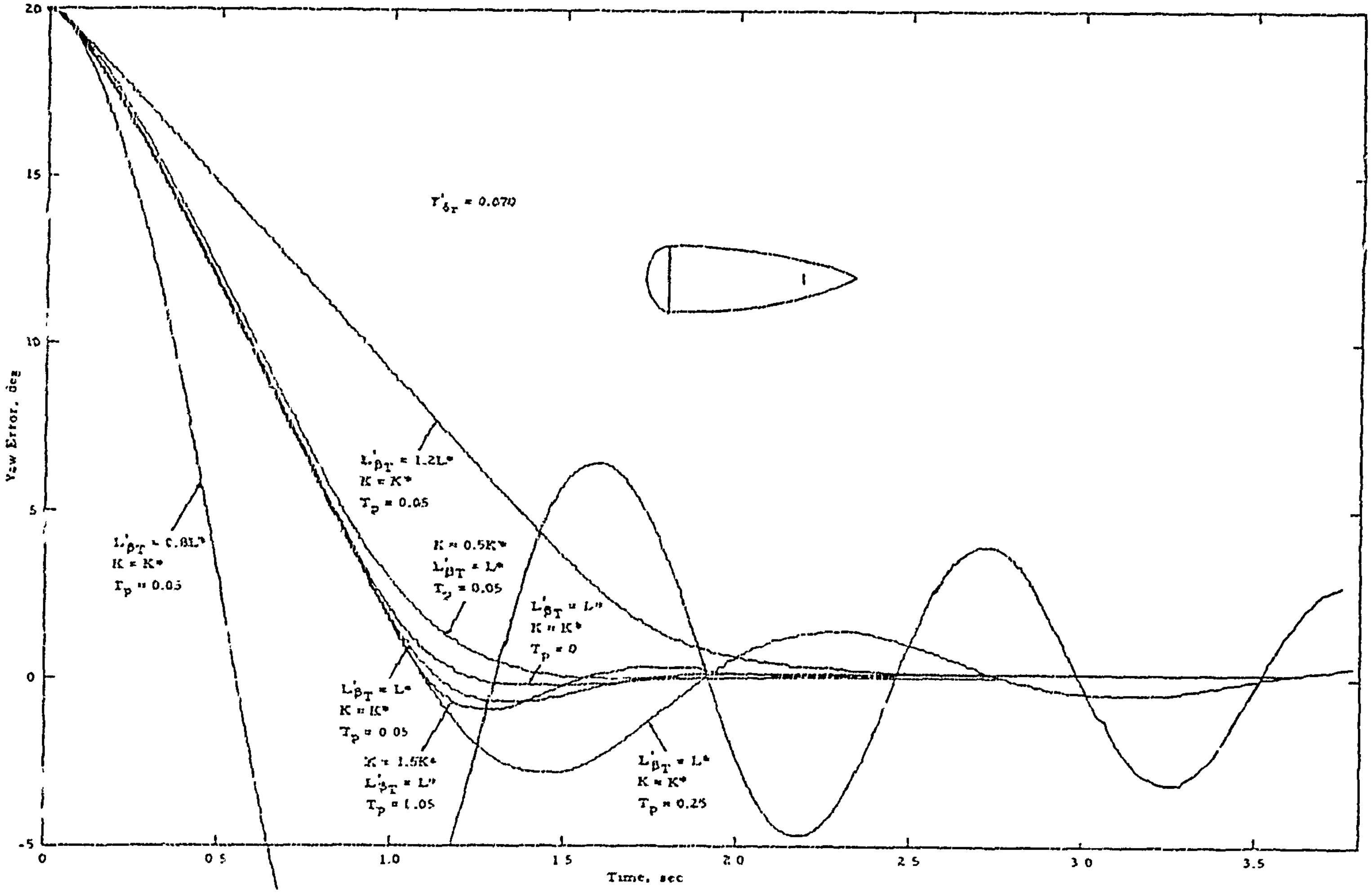


FIG. 3. Response of Body 1, Including Effect of Time Lag (T_p) and Changes From the Design Criteria.

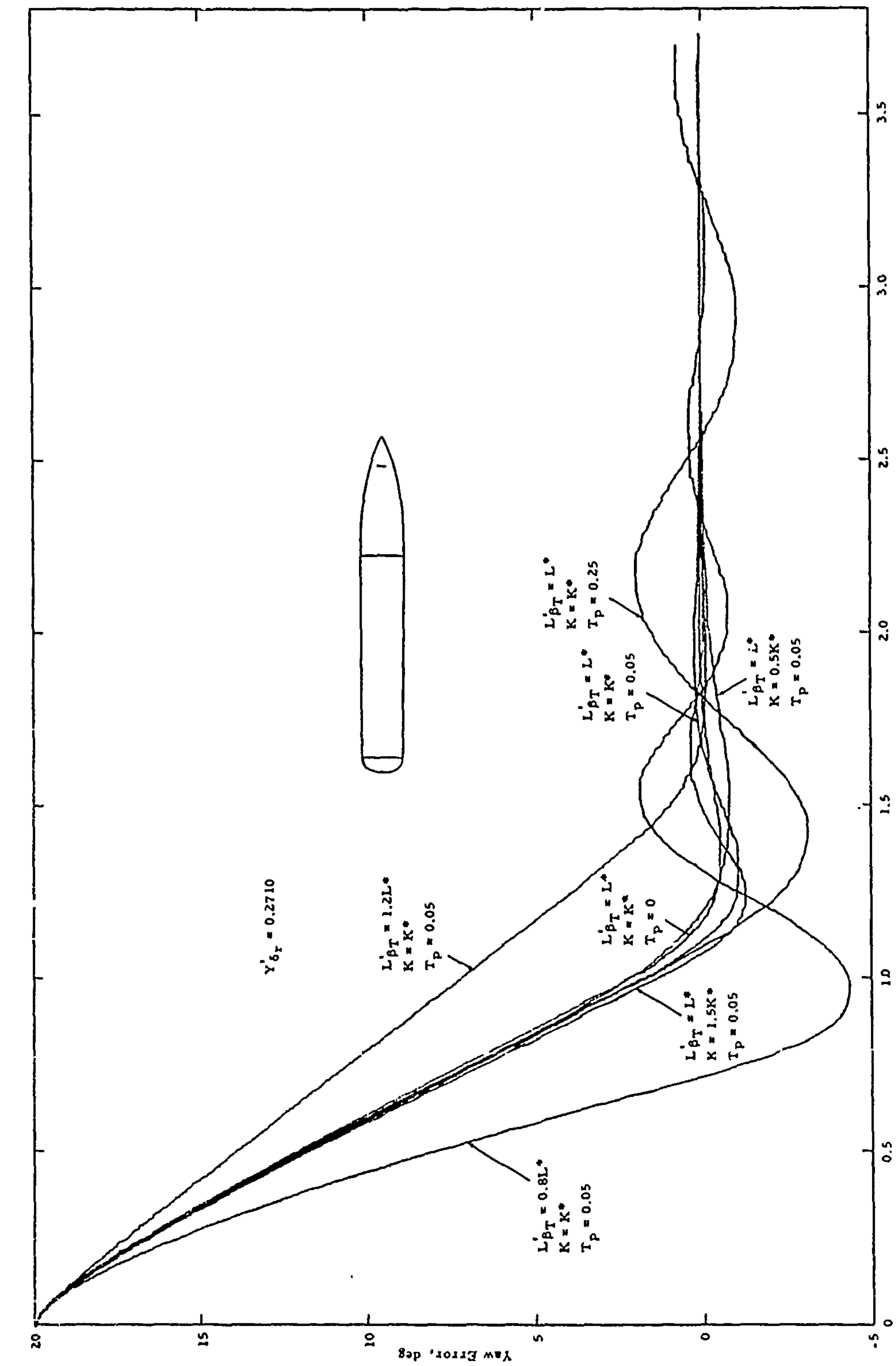


FIG. 4. Response of Body 1.1, Including Effect of Time Lag (T_p) and Changes From the Design Criteria.

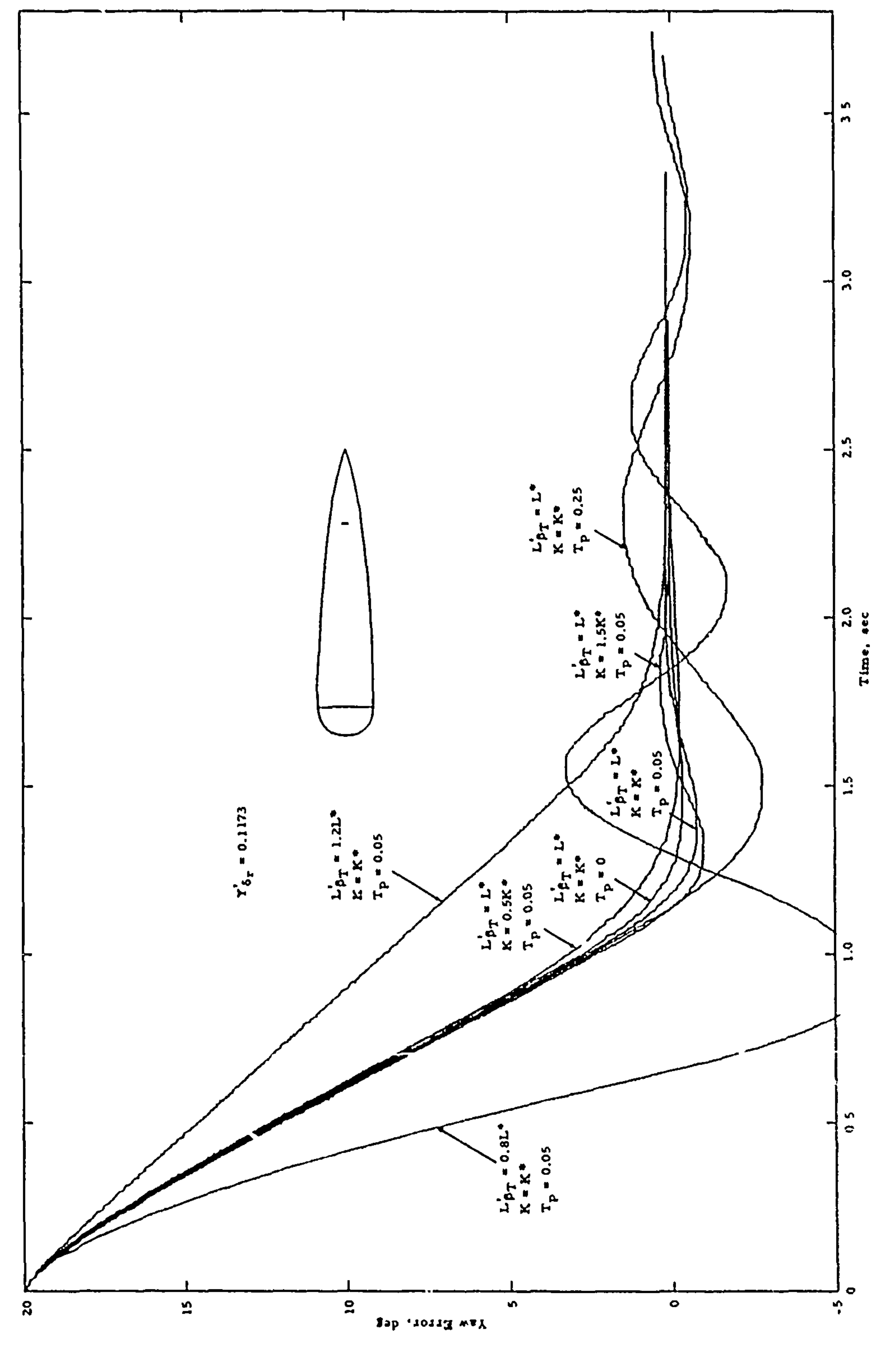


FIG. 5. Response of Body 2, Including Effect of Time Lag (Tp) and Changes From the Design Criteria.

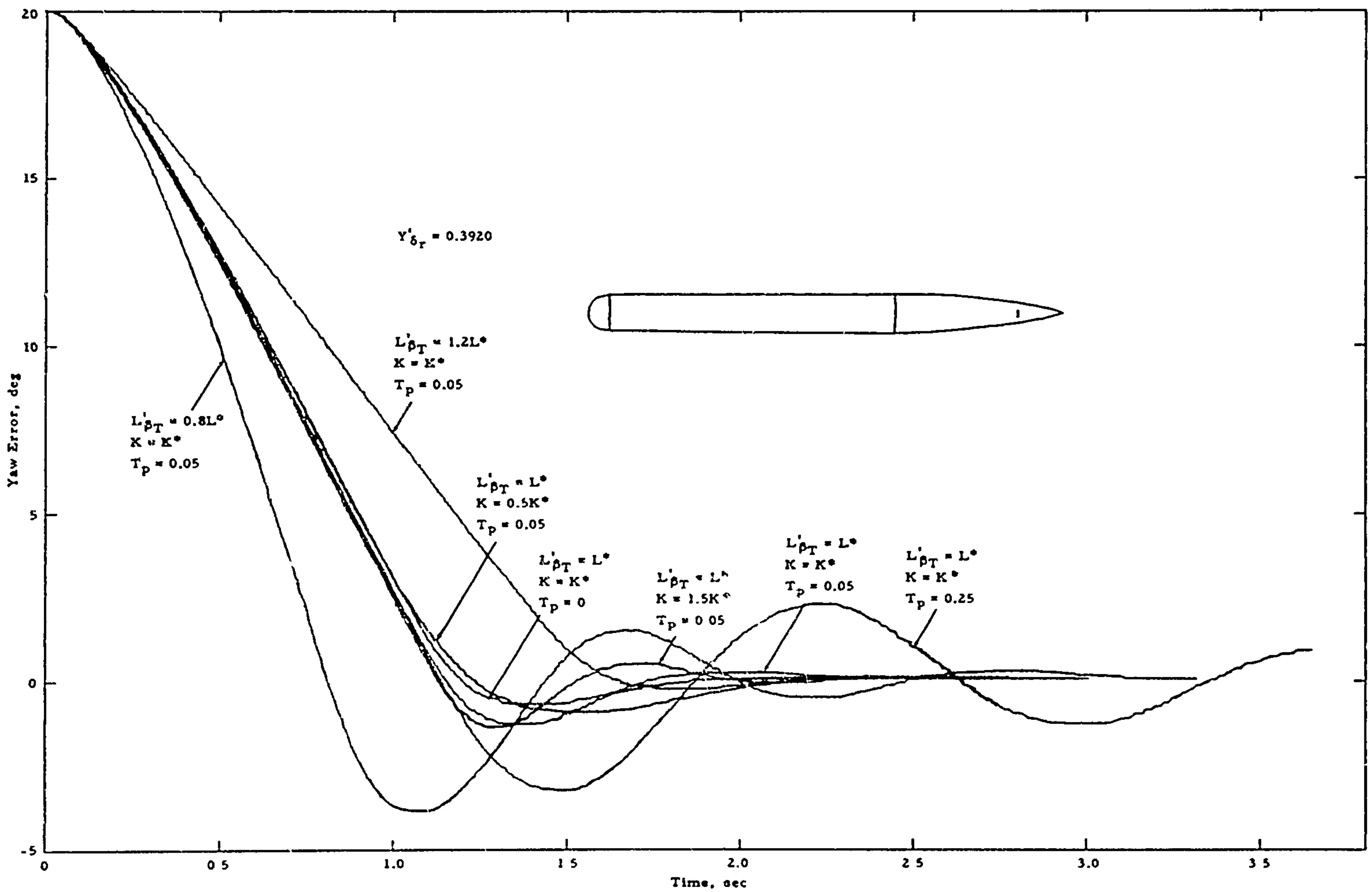


FIG. 6. Response of Body 2.1, Including Effect of Time Lag (T_p) and Changes From the Design Criteria.

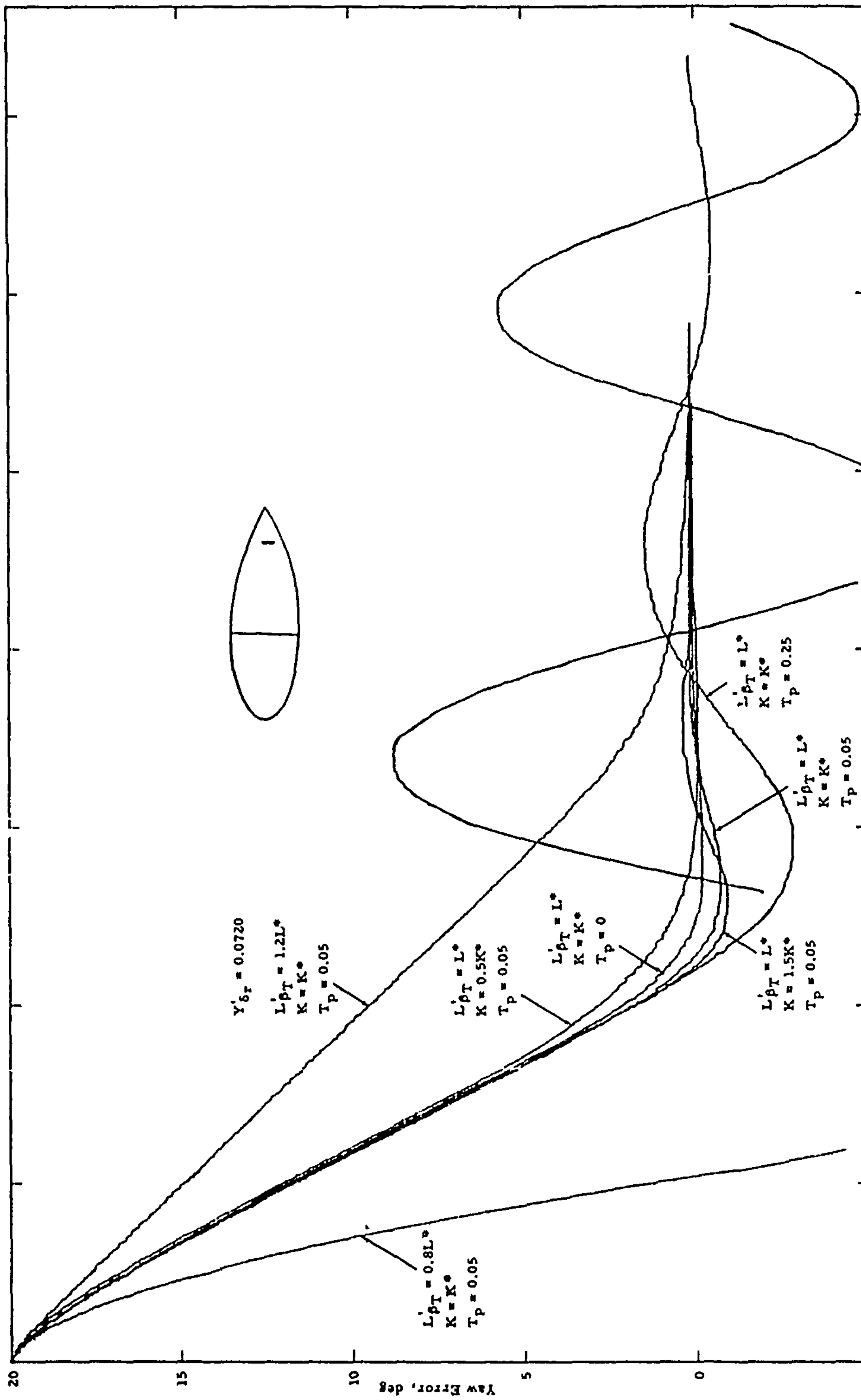


FIG. 7. Response of Body 3, Including Effect of Time Lag (T_p) and Changes From the Design Criteria.

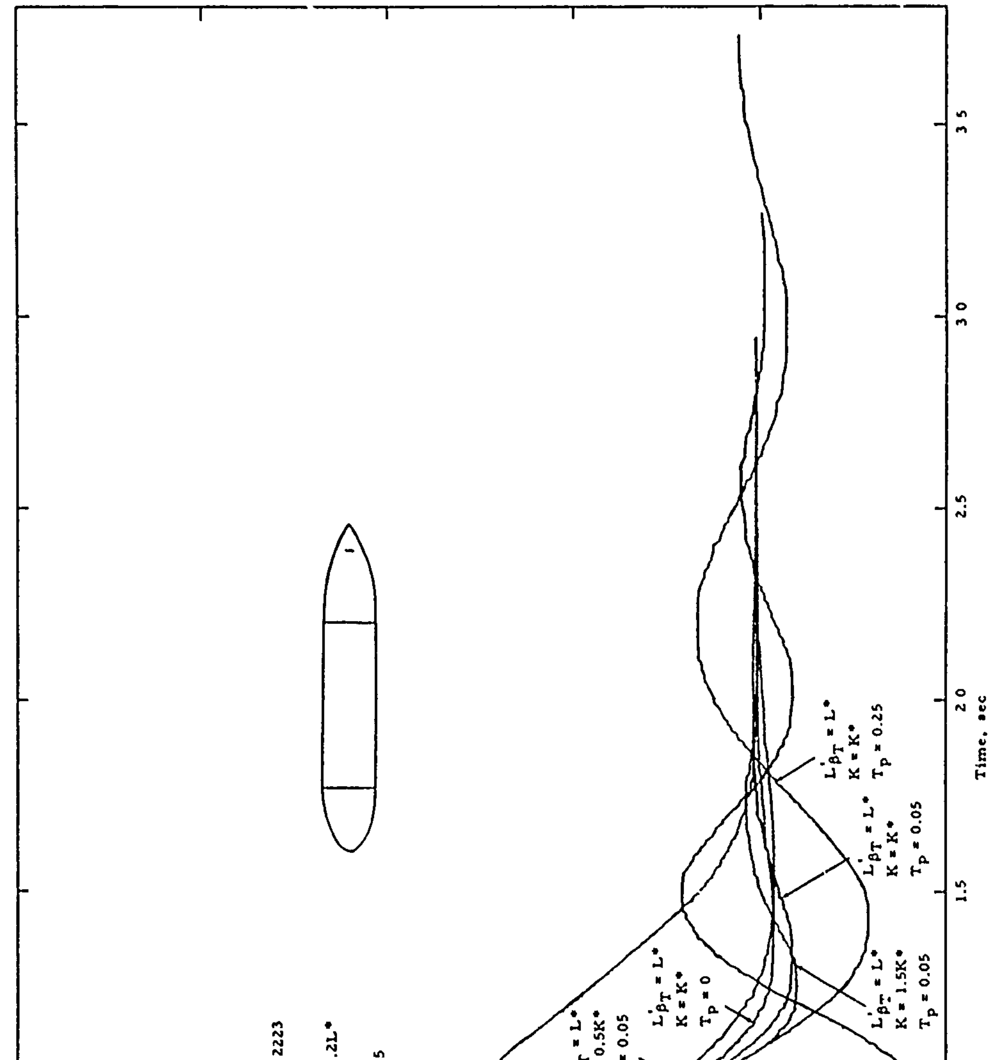


FIG 8. Response of Body 3.1, Including Effect of Time Lag (T_p) and Changes From the Design Criteria.

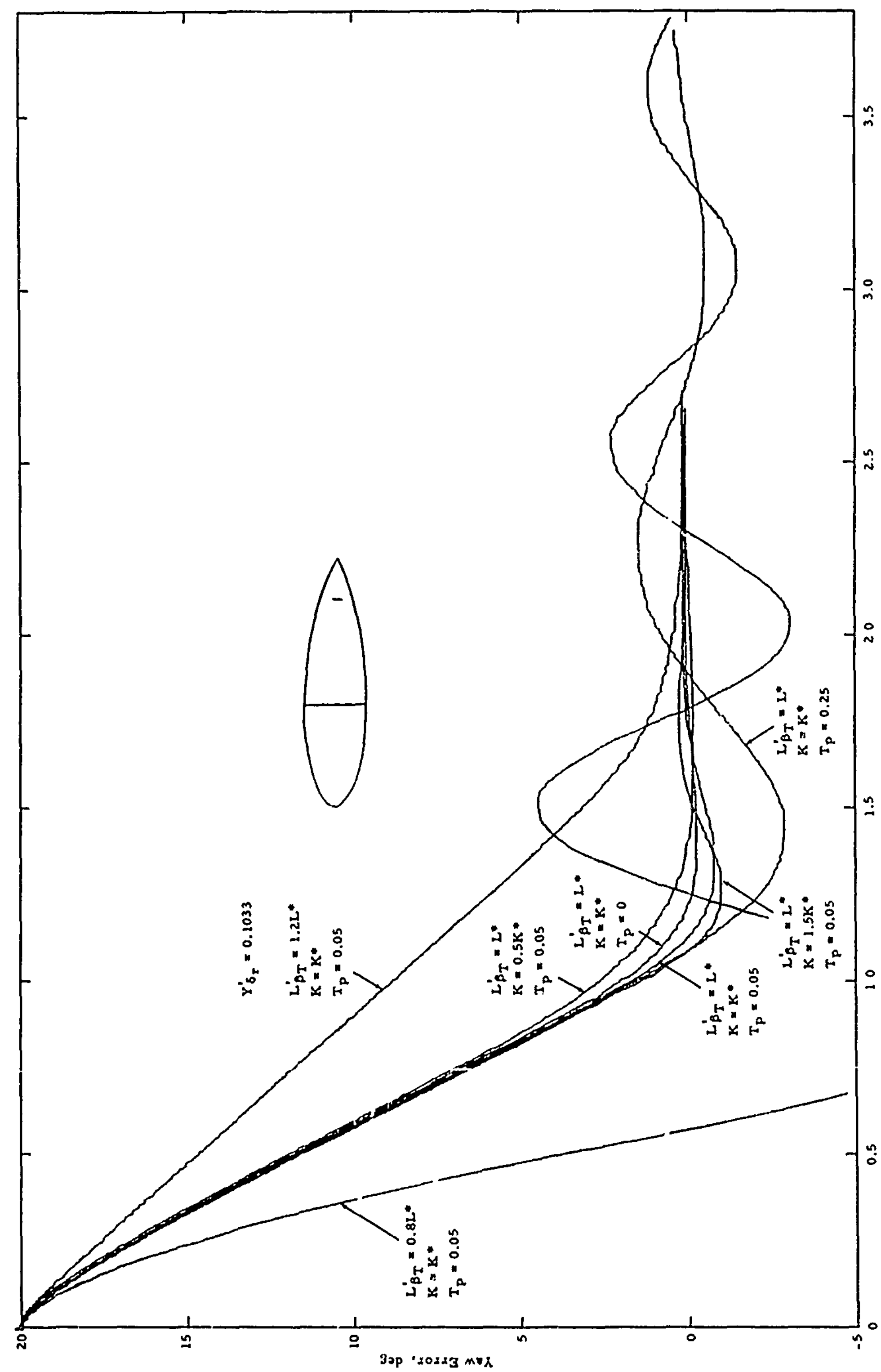


FIG. 9. Response of Body 4, Including Effect of Time Lag (T_p) and Changes From the Design Criteria.

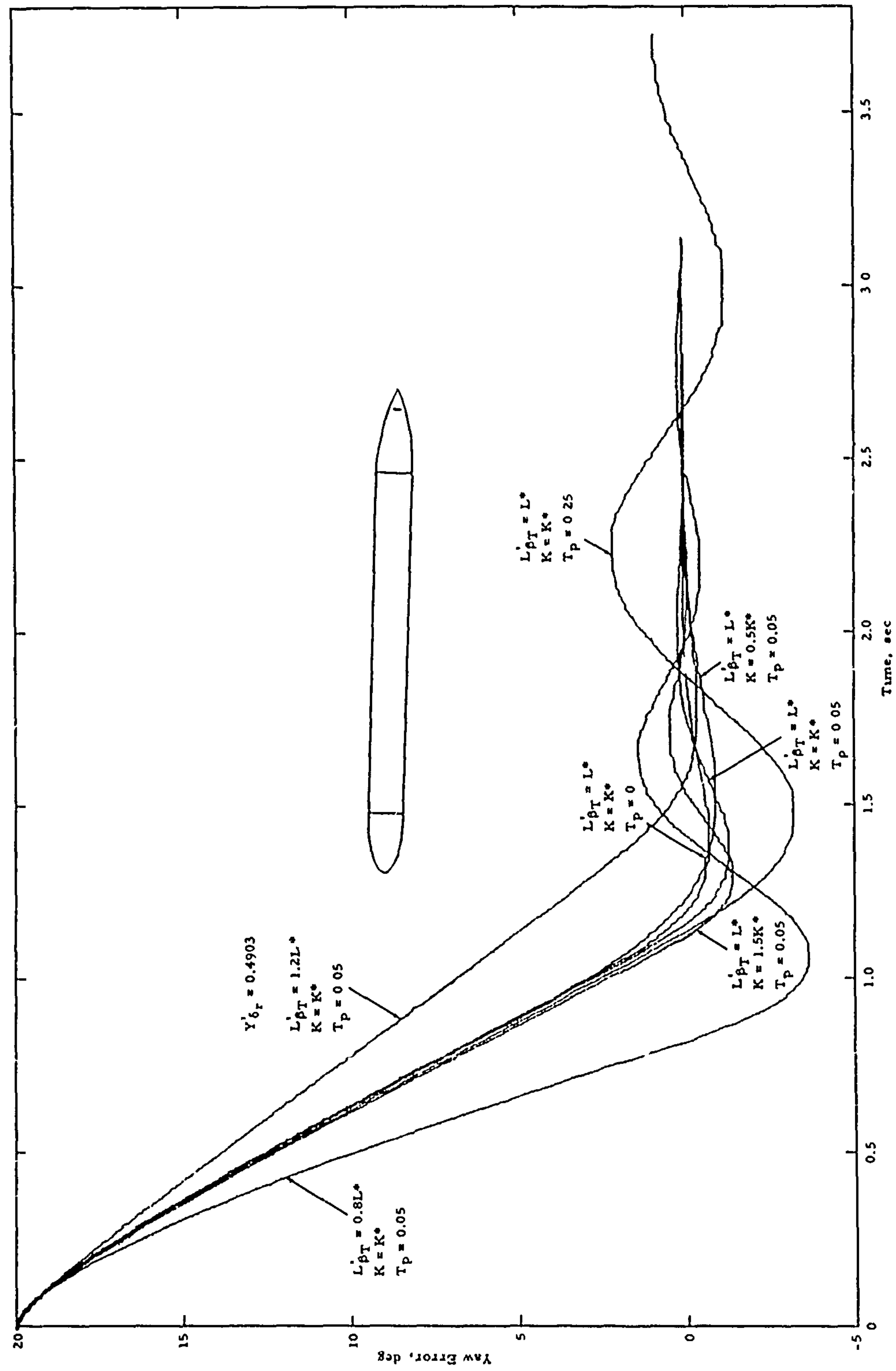


FIG. 10. Response of Body 4.1, Including Effect of Time Lag (Tp) and Changes From the Design Criteria.

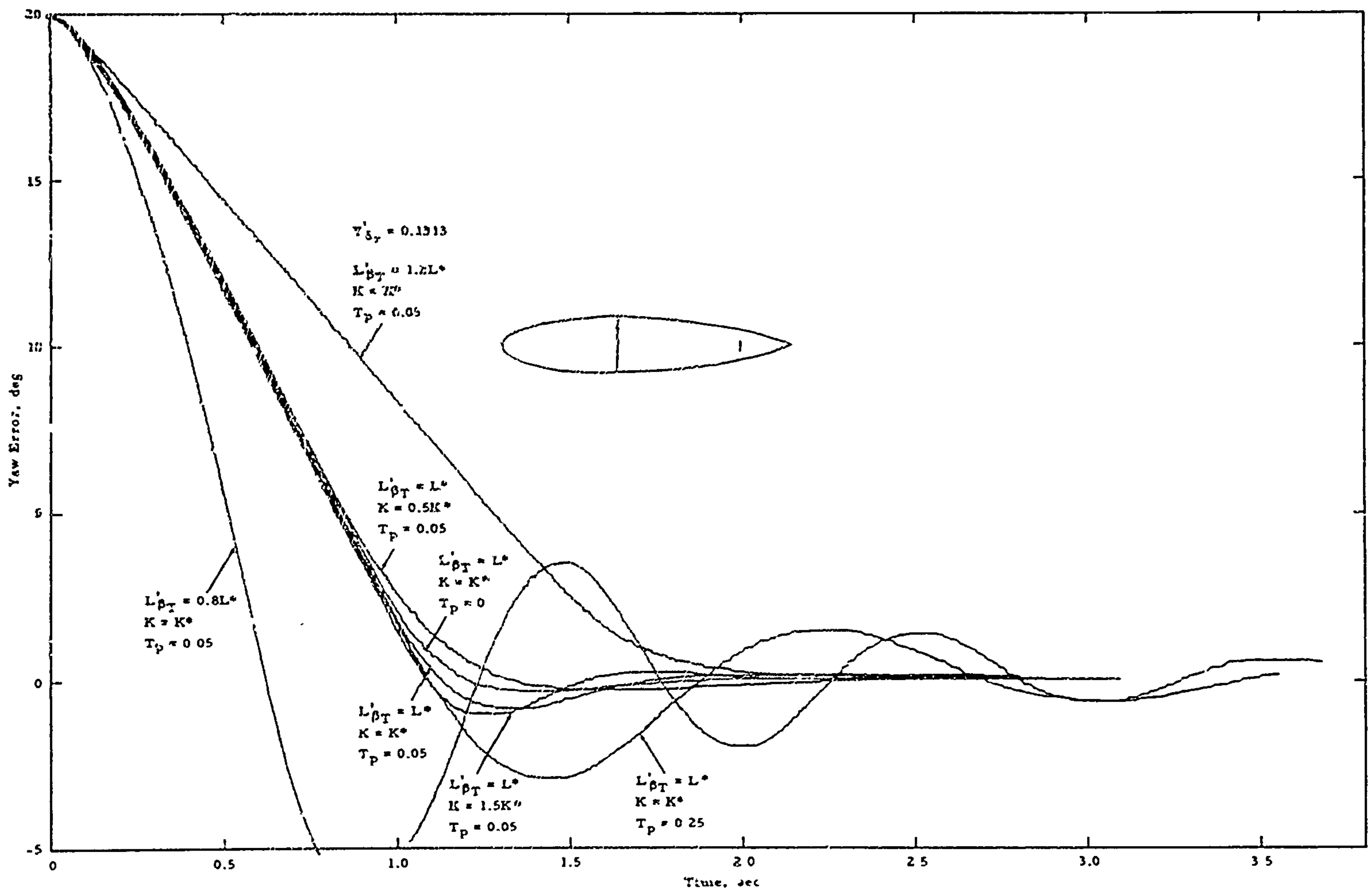


FIG. 11. Response of Body 5, Including Effect of Time Lag (T_p) and Changes From the Design Criteria.

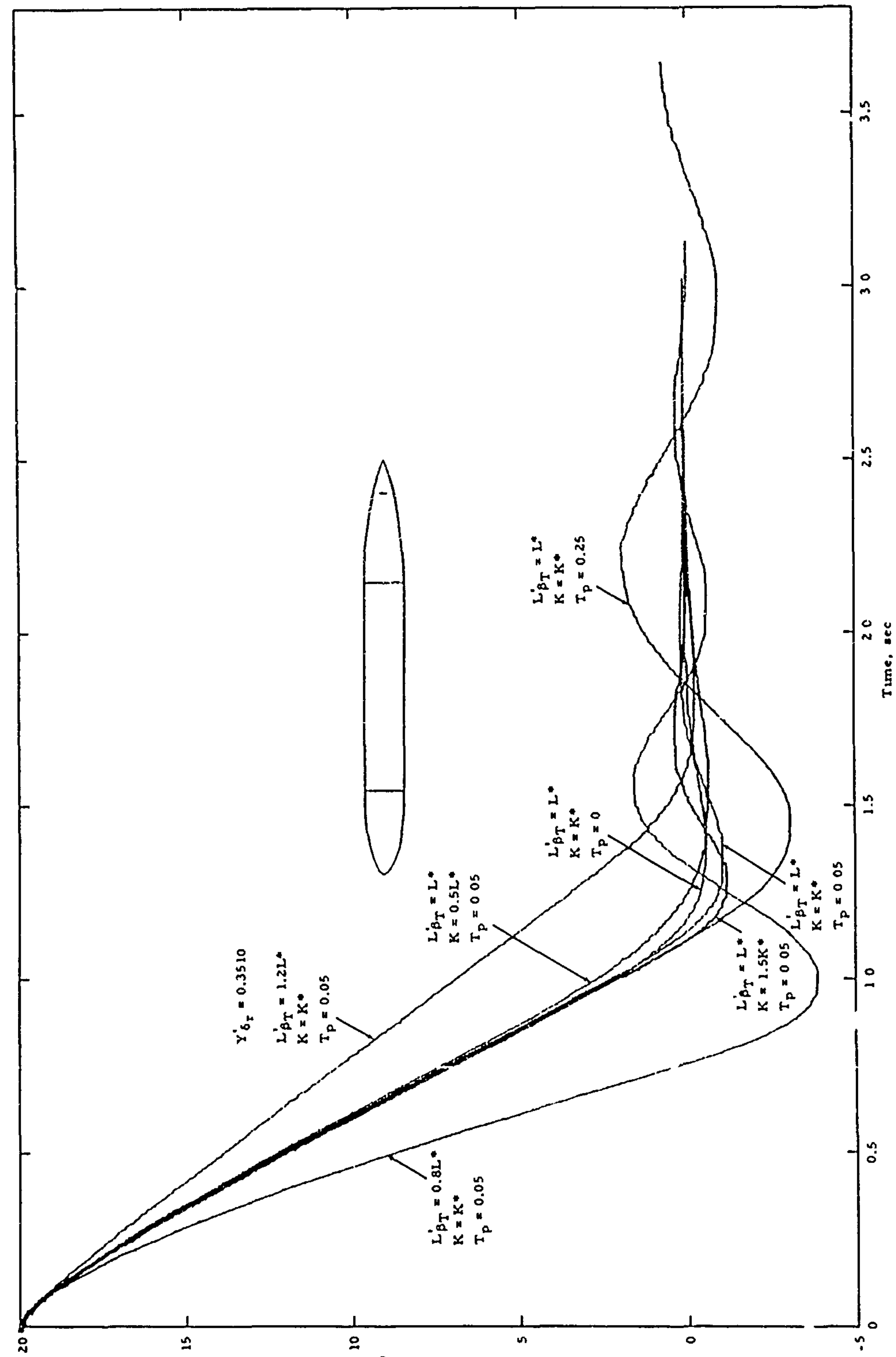


FIG. 12. Response of Body 5.1, Including Effect of Time Lag (T_p) and Changes From the Design Criteria.

section. The question arises as to how changes in the assumptions would affect the criteria and the response. Table 3 shows the results of independent changes of Body 1.1 in weight, volume, speed, and location of the center of tail lift (i.e., x_ρ/ℓ). The changes are specified in the table and values L*, K*, and $-Y'_{\delta_r}/(\dot{\psi}_{ss}/\delta_{r_o})$ are given. Figure 13 shows the correction of a 20-degree yaw error for each of these cases; the obvious conclusion is that the response is not affected as long as the dimensionless tail-lift coefficient and the constant K from the control equation are given by the criteria L'_{β_T} = L* and K = K* (assuming the same steady-state turn rate).

It might also be important to discover the effect of replacing the assumption of a 5 ft^3 volume by the assumption of a constant cross-sectional area. Table 4 shows that the addition of a cylindrical section requires a larger L*, -K*, and $-Y'_{\delta_r}/(\dot{\psi}_{ss}/\delta_{r_o})$. The same is true, although to a lesser degree, if the point of maximum diameter is moved rearward, without addition of a cylindrical section.

EFFECT OF CRITERIA CHANGES ON RESPONSE

The effect of separate changes in L'_{β_T} and K on the response of each body is also investigated in Fig. 3 through 12. For the recommended L'_{β_T} = L* and a time lag of 0.05 second, K was given the two values 1.5K* and 0.5K*. The response for each of these values does not vary significantly from that obtained with K = K*. However, holding K = K* fixed and varying L'_{β_T} leads to quite significant changes. A 20% increase in L'_{β_T} (to L'_{β_T} = 1.2L*) results in slower correction of a yaw error, while a 20% decrease (to L'_{β_T} = 0.8L*) leads to large-amplitude oscillations. The effects are more pronounced on bodies without a cylindrical section. Figure 14 shows that at these changed

TABLE 3. Effect of Changes in Weight, Speed, Volume, and Location of Tail Lift for Body 1.1

Body	Description of Change	L*	Change, %	$\frac{-Y'_{\delta_r}}{\dot{\psi}_{ss}/\delta_{r_o}}$	Change, %	-K*	Change, %
1.1	. . . . .	1.84	. . .	0.0819	. . .	0.270	. . .
1.11	Specific gravity = 1	1.74	-5.4	0.0774	-5.5	0.239	-11.5
1.12	Specific gravity = 1.4	1.92	+4.3	0.0846	+3.3	0.295	+9.3
1.13	V = 30 knots or $\mathcal{V} = 2.37 \cdot 5$ ft^3	1.85	+0.5	0.1104	+34.8	0.408	+51.1
1.14	V = 50 knots or $\mathcal{V} = 0.512 \cdot 5$ ft^3	1.80	-2.2	0.0615	-24.9	0.185	-31.5
1.15	$\frac{x_p}{\ell}$ = 0.9 that of Body 1.1	2.08	+13.0	0.0831	+1.5	0.237	-12.2
1.16	$\frac{x_p}{\ell}$ = 1.1 that of Body 1.1	1.65	-10.3	0.0762	-7.0	0.303	+12.2

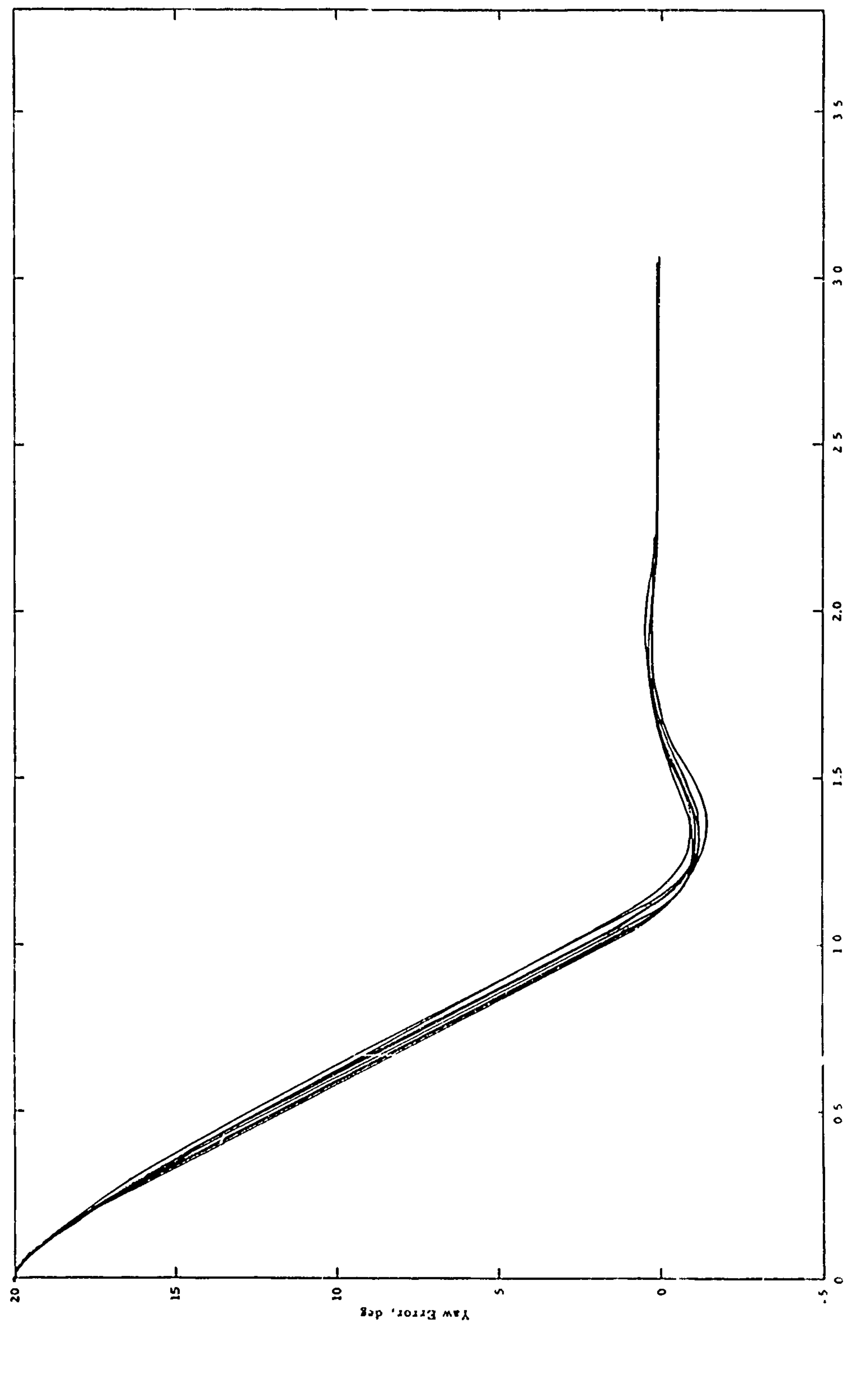


FIG. 13. Effect of Change in Weight, Speed, Volume, and Location of Tail Lift on the Correction of a Yaw Error (See Table 3).

TABLE 4. Effect of Change in Body Shape if Cross-Sectional Area Is Kept Constant

Body Description	L^*	$\frac{-Y'_{\delta_r}}{\dot{\psi}_{ss}/\delta_{r_o}}$	$-K^*$
Body 2 $\frac{\ell_n}{\ell_n + \ell_t} = 0.1$, $\frac{\ell_c}{\ell} = 0$, $\frac{\ell_n + \ell_t}{d} = 5$, $\ell = 6.63$ ft	1.31	0.0352	0.079
Body 2 with cylindrical section added so that $\frac{\ell_c}{\ell} = 0.6$	1.94	0.2181	0.673
Same cross-sectional area as Body 2 $\frac{\ell_n}{\ell_n + \ell_t} = 0.4$, $\frac{\ell_c}{\ell} = 0$, $\frac{\ell_n + \ell_t}{d} = 5$, $\ell = 6.63$ ft	1.66	0.0412	0.089

values of L'_{β_T} the response cannot be improved significantly by varying K. This does not mean, however, that L^* is an optimum value; the explanation can be found in Expression 11 and Eq. 13. In Eq. 13 the steady-state turn rate is directly proportional to Y'_{δ_r} and inversely proportional to a linear, increasing function

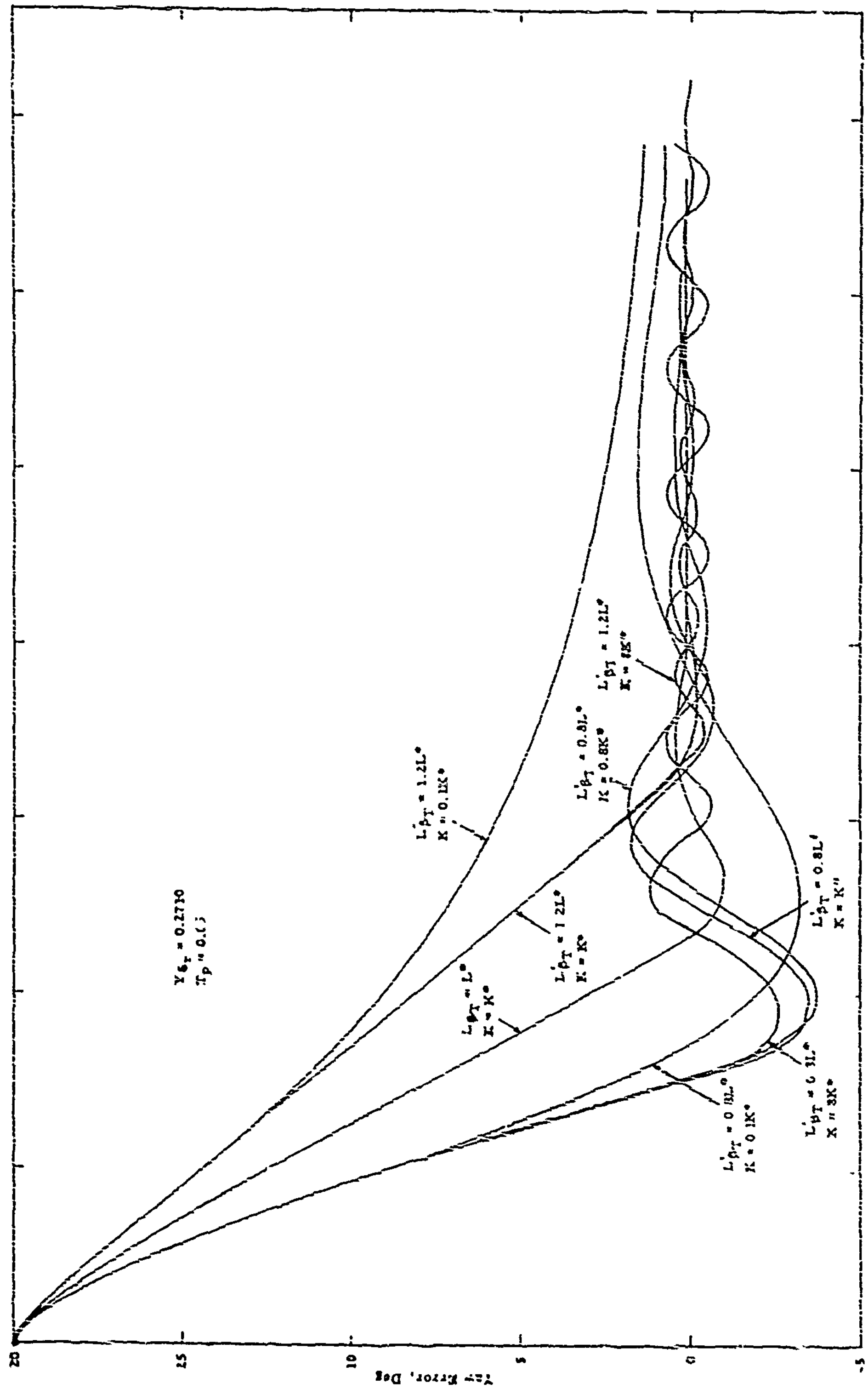


FIG. 14. Response of Body 1.1 Showing Effect of Changes in Both L'_{β_T} and K.

of L'_{β_T}. Thus, if L'_{β_T} is reduced to $0.8L^*$ and Y'_{δ_r} is not changed, the steady-state turn rate is increased, which accounts for the faster initial response. Also, since the band of proportional control has not changed (Expression 11), the turn rate is sufficiently high to cause the torpedo to overshoot and the rudders to become hard-over at -6 degrees. In fact, the overshoot is so great that the torpedo has time to build up a sufficiently large turn rate in the opposite direction to again overshoot the proportional band, which results in the extreme oscillation. In the case of $L'_{\beta_T} = 1.2L^*$, with no change in Y'_{δ_r}, the turn rate is decreased so that no overshoot occurs, but the time needed to correct the error increases. If Y'_{δ_r} is reduced for $L'_{\beta_T} = 0.8L^*$ and increased for $L'_{\beta_T} = 1.2L^*$, so as to maintain the original steady-state turn rate (Fig. 3 through 12), the initial part of the curves will be approximately the same, differing only because of different lengths of time required to reach the steady-state turn rate. In the former case ($L'_{\beta_T} = 0.8L^*$ and a reduced Y'_{δ_r}) the band of proportional control is narrowed and the danger of oscillation by overshooting the band persists. K may be decreased to widen the band, but then both L'_{β_T} and K differ from L^* and K^* and the corresponding frequency and damping (ω and ζ) may be undesirable. In the latter case ($L'_{\beta_T} = 1.2L^*$ and an increased Y'_{δ_r}) the band of proportional control is widened so that there should be no danger of overshooting the band. In fact, K may be increased to reduce the proportional band width.

Figures 15 and 16 indicate how ω and ζ change with L'_{β_T} and K. The curves were plotted for Body 1.1 only, but corresponding changes on the other bodies should result in similar curves. The principal conclusions are that L'_{β_T} is largely independent of ω and depends primarily on ζ, while K is independent of ζ and depends mainly on ω. Also, L'_{β_T} increases with ζ and -K increases with ω.

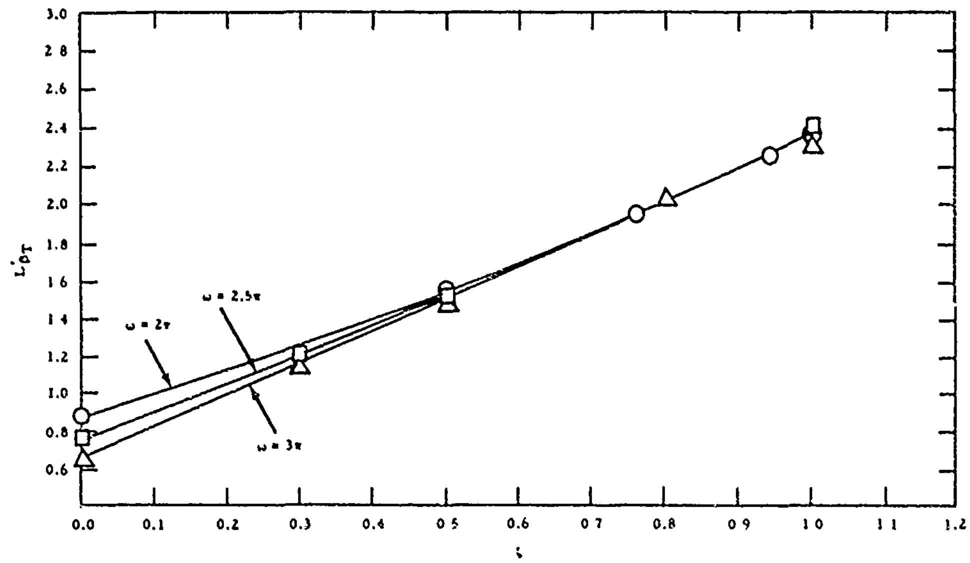


FIG. 15. Dependence of Dimensionless Tail-Lift Coefficient on Natural Frequency and Damping (Body 1.1).

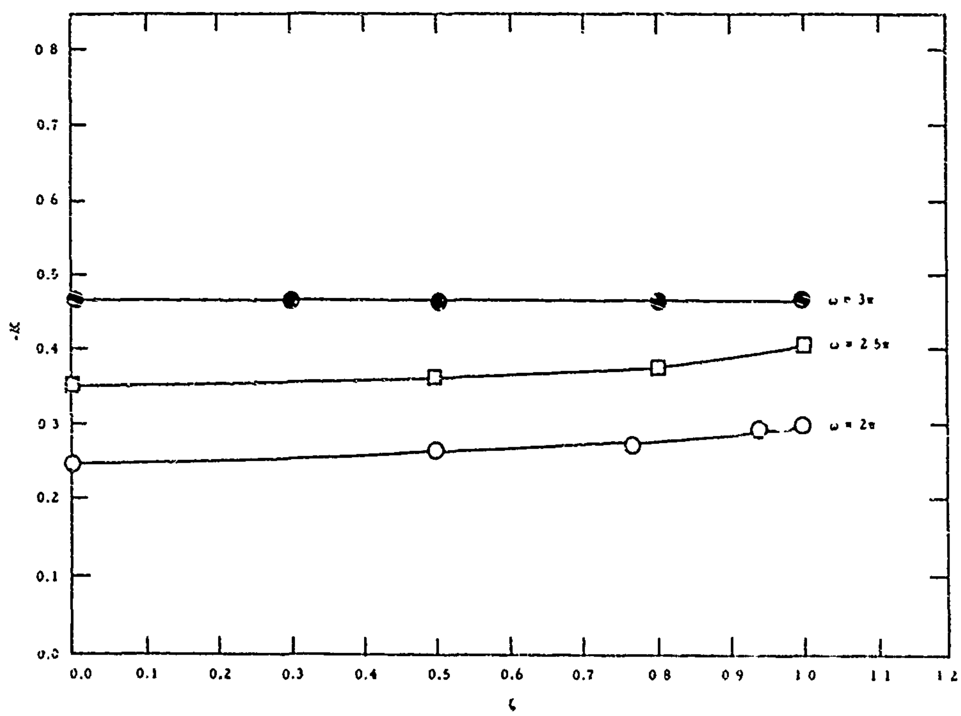


FIG. 16. Dependence of Control Constant on Natural Frequency and Damping (Body 1.1).

RESPONSE OF SAMPLE TORPEDOES TO CONSTANT RUDDER DEFLECTION

Figure 17 shows how each of the ten bodies approaches a 20 deg/sec steady-state turn rate for $L'_{\beta_T} = L^*$ and a rudder deflection of -6 degrees. The rudders were assumed to deflect from 0 to -6 degrees in 0.05 second at a constant rate. The curves do not differ greatly, further demonstrating the constancy of response with respect to the criteria, and Body 1.1 gives the central curve. Figure 18 shows, for Body 1.1, the effect of a ±20% change in L'_{β_T} [i.e., $L'_{\beta_T} = (1 \pm 0.2)L^*$], with a corresponding change in Y'_{δ_r} to give the same steady-state turn rate. A change of approximately 5% in L* would make all the curves of Fig. 17 practically identical.

Because of its importance in determining cavitation resistance, the angle of attack at the tail is plotted in Fig. 19 for each of the ten bodies.

APPROXIMATE FORMULAS FOR DESIGN CRITERIA

The following approximate formulas for L*, K*, and $-Y'_{\delta_r}/(\dot{\psi}_{ss}/\delta_{r_0})$, in terms of the body parameters $\eta = \ell_n/(\ell_n + \ell_t)$, ℓ_c/ℓ, and $\ell_p/d = (\ell_n + \ell_t)/d$, should give sufficiently accurate values for design purposes:

$$(14) \qquad L^* \simeq (0.909 - 0.37\eta)\frac{\ell_c}{\ell} + (\eta + 1.205)$$

$$(15) \qquad -K^* \simeq \left(\frac{10}{3}\eta - \frac{1}{3}\right)$$

$$\left[\left(0.314\frac{\ell_p}{d} - 0.013\right)\left(\frac{\ell_c}{\ell}\right)^2 - \left(0.15\frac{\ell_p}{d} + 0.103\right)\frac{\ell_c}{\ell} + \left(0.003\frac{\ell_p}{d} - 0.008\right)\right]$$

$$+ 0.019\frac{\ell_p}{d} - 0.016 + \left(0.1\frac{\ell_p}{d} + 0.082\right)\frac{\ell_c}{\ell}$$

and

$$(16)\quad \frac{-Y'_{\delta_r}}{\dot{\psi}_{ss}/\delta_{r_o}} \simeq \left(\frac{10}{3}\eta - \frac{1}{3}\right)$$

$$\cdot\left[\left(0.052\frac{\ell_p}{d} + 0.067\right)\left(\frac{\ell_c}{\ell}\right)^2 - \left(0.0235\frac{\ell_p}{d} + 0.0512\right)\frac{\ell_c}{\ell} + \left(0.0019\frac{\ell_p}{d} - 0.0051\right)\right]$$

$$+ 0.007\frac{\ell_p}{d} + \left(0.0185\frac{\ell_p}{d} + 0.0448\right)\frac{\ell_c}{\ell}$$

These approximations save considerable computing time with quite satisfactory accuracy. Table 5 compares the approximate values for the ten bodies under consideration with the exact values. Two other bodies, with parameters η, ℓ_c/ℓ, and ℓ_p/d lying within the range of the ten sample bodies, were chosen at random. The approximate formulas hold, the largest error being 11% for K*.

These approximations were established under the assumptions of the section "Parameters of Sample Torpedoes" and should therefore be used with caution if a particular torpedo departs significantly from any of the assumptions. Table 3 shows the effect of changes in certain of the assumptions for one of the sample bodies; if the approximate formulas are used with corrections as indicated in this table, good approximations should still result.

CONCLUSIONS

It has been shown that all torpedoes of conventional form, designed to the criteria L* and K*, will have practically identical responses both for the constant-rudder case and for linear proportional control. The intention was to obtain a reasonably adequate response rather than an "optimum" response. In fact, the term "optimum" itself is questionable; it is not at all certain that a response which corrects an error in the shortest time,

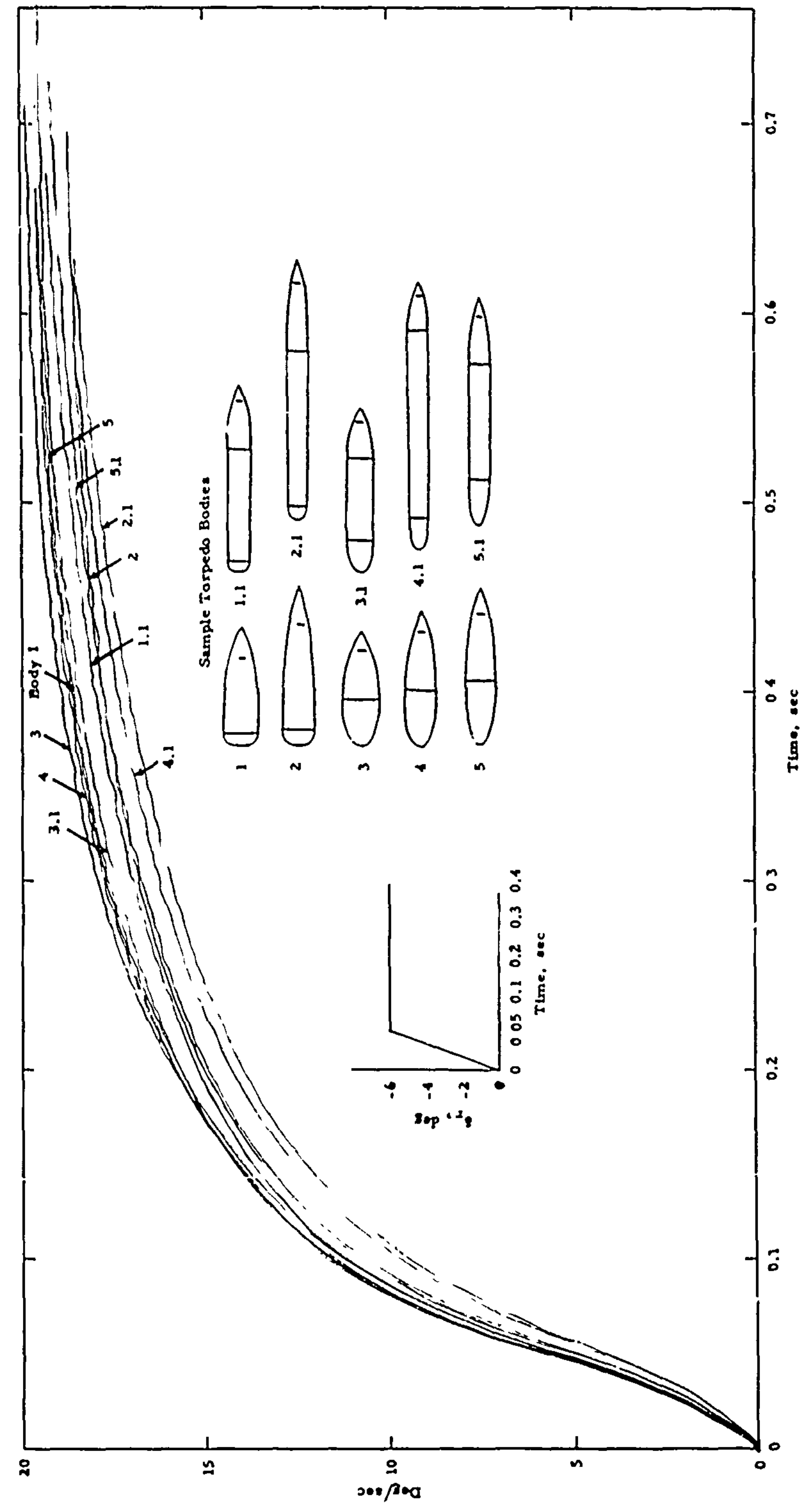


FIG. 17. Response of Ten Sample Bodies to Constant Rudder Deflection.

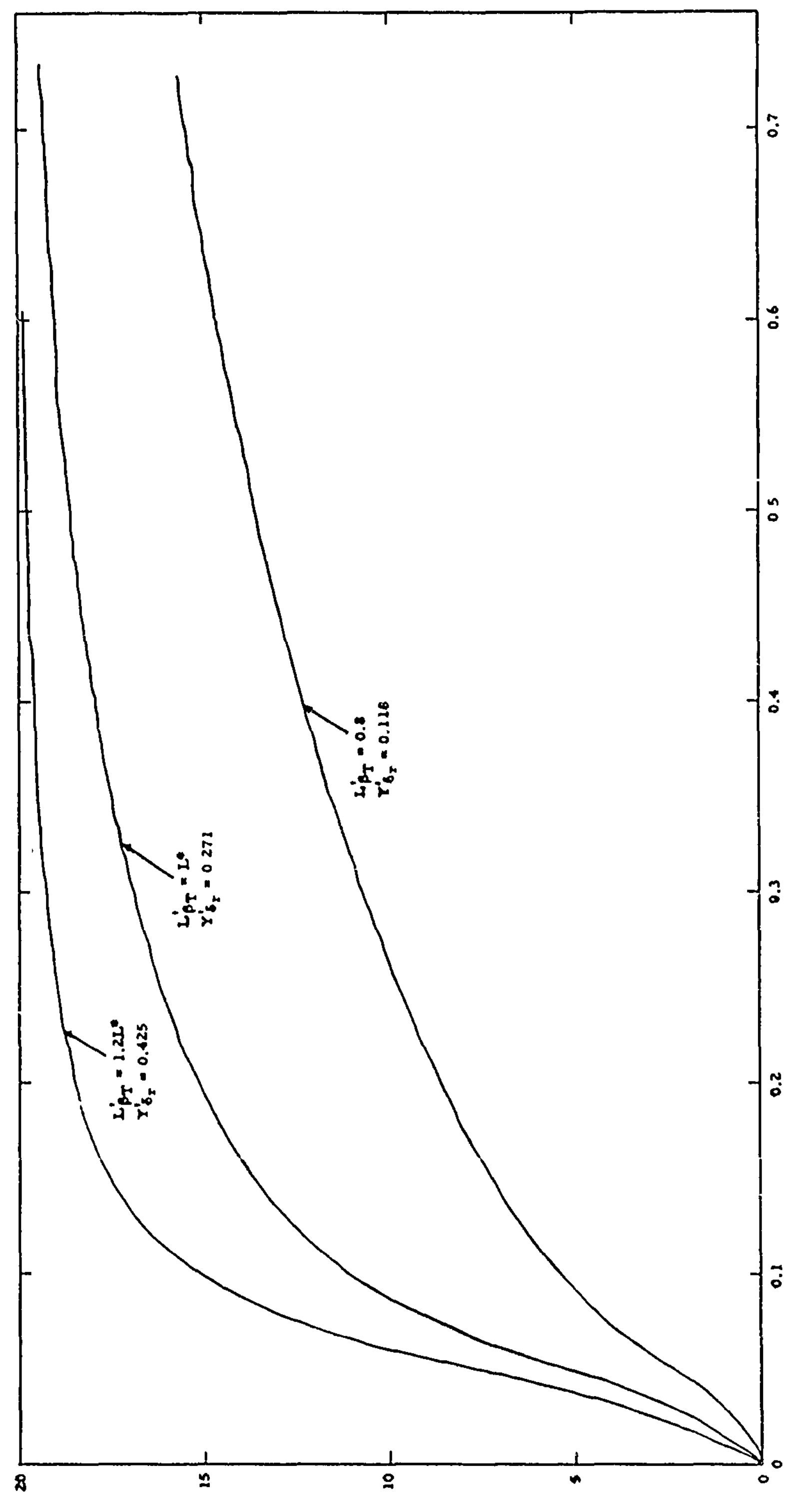


FIG. 18. Effect of Changes in Tail Size and Rudder Size on Response of Body 1.1 to Constant Rudder Deflection.

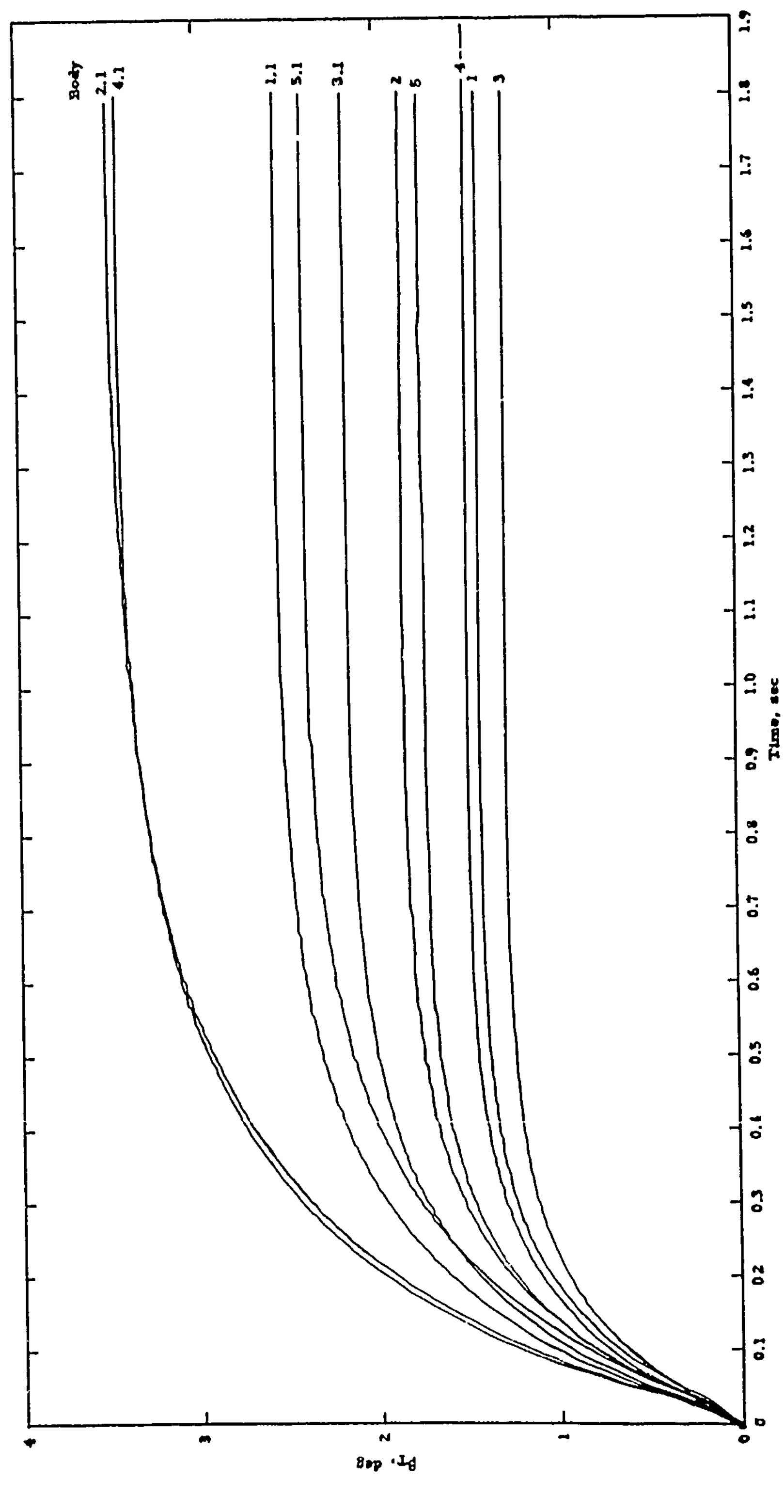


FIG. 19. Angle of Attack at the Tail for Each of the Ten Sample Bodies. Steady-state turn rate = -20 deg/sec.

TABLE 5. Comparison of Exact and Approximate Formulas

Body	L^*			$-K^*$			$\frac{-Y'_{\delta_r}}{\dot{\psi}_{ss}/\delta_{r_o}}$		
	Exact	Approx.	Error, %	Exact	Approx.	Error, %	Exact	Approx.	Error, %
1	1.30	1.31	+0.8	0.041	0.041	0	0.0211	0.021	-0.5
1.1	1.84	1.83	-0.5	0.270	0.270	0	0.0819	0.081	-1.1
2	1.31	1.31	0.0	0.079	0.079	0	0.0352	0.035	-0.6
2.1	1.81	1.83	+1.1	0.428	0.428	0	0.1176	0.117	-0.5
3	1.56	1.61	+3.2	0.042	0.042	0	0.0216	0.0216	0.0
3.1	1.93	1.99	+3.1	0.189	0.188	-0.5	0.0667	0.0667	0.0
4	1.63	1.61	-1.2	0.064	0.064	0	0.0310	0.0305	-1.6
4.1	2.06	2.14	+3.9	0.513	0.513	0	0.1471	0.1468	-0.2
5	1.65	1.61	-2.4	0.086	0.086	0	0.0394	0.0394	0.0
5.1	2.04	1.99	-2.5	0.332	0.332	0	0.1053	0.1055	+0.2
Example 1: $\eta = 0.2$, $\ell_p/d = 3.5$, $\ell_c/\ell = 0.5$	1.84	1.82	-1.1	0.231	0.253	+11.0	0.0744	0.0783	+5.2
Example 2: $\eta = 0.2$, $\ell_p/d = 4$, $\ell_c/\ell = 0.6$	1.87	1.91	+2.1	0.358	0.359	+0.3	0.1022	0.104	+1.8

regardless of the amount of overshoot and oscillation, is preferable to one which takes longer but results in less overshoot and oscillation. This question cannot be answered in general terms. It can be answered only for a specific torpedo, after physical restrictions and performance requirements are known from computer and weapons analysis studies. Past experience indicates, however, that the response of a torpedo with a dimensionless tail-lift coefficient L* is quite adequate, and certainly adequate for preliminary design.

It should be noted that the quantity K* is uniquely associated with the linear proportional control system and hence has no meaning for other control systems. However, consideration of such a linear proportional control system in this study has served the following important purposes: It has lead to values of the dimensionless tail-lift coefficient L* which result in torpedoes with an adequate degree of stability and approximately equal response to a constant rudder deflection. Further it has shown that torpedoes having $L'_{\beta_T} = L^*$ can be controlled by a linear proportional control system and hence will probably behave well under other control systems. Since any motion of the control surface can be approximated by a succession of constant rudder deflections, and since the torpedoes respond similarly with a constant (step) deflection, it should be possible to make them respond similarly with any type of control.

Some of the more important conclusions can be summarized.

1. Bodies of Equal Volume. An elongated streamlined body, or a body with a cylindrical section, has a smaller steady-state turn rate per degree of rudder deflection than a shorter streamlined body, if tail and rudder sizes are equal; to obtain equal response from the elongated body the size of the tail must be decreased and the size of the rudder increased.

Moving the maximum diameter of a streamlined body rearward increases the steady-state turn rate per degree of rudder deflection; to obtain equal response, the size of the tail must be increased, but the size of the rudder can remain practically unchanged.

2. Bodies of Equal Maximum Cross-Sectional Area. The addition of a cylindrical section decreases the steady-state turn rate per degree of rudder deflection, if tail and rudder size are equal; to obtain equal response, the size of both tail and rudder must be increased.

Moving the maximum diameter rearward increases the steady-state turn rate per degree of rudder deflection; to obtain equal response, the size of both tail and rudder must be increased.

3. The steady-state turn rate per degree of rudder deflection is a decreasing function of tail size and an increasing function of rudder size.

4. If the steady-state turn rate per degree of rudder deflection is held constant, a torpedo with larger rudder and tail will respond more quickly than a torpedo with smaller rudder and tail.

5. If the steady-state turn rate per degree of rudder deflection is held constant by adjusting the size of the rudder, small changes in torpedo weight, volume, speed, and center of gravity have little effect on response, provided the tail lift is changed according to the criterion developed in this study.

6. In general, a time lag of 0.05 second in the control system appears to be acceptable, but a time lag of 0.25 second apparently is not.

7. According to Ref. 1 geometrically similar bodies have equal dimensionless body coefficients. If the bodies also are of uniform density with the same specific gravity (so that

the dimensionless masses and moments of inertia are equal), and if the centers of tail and rudder lift are located so that x_ρ/ℓ and x_δ/ℓ remain constant, the bodies will have identical responses for both linear proportional control and step rudder deflections as long as the ratio V/ℓ is constant. This is due to the fact that the constants a, b, c, d, e, and f (Appendix A) are then equal and hence the equations of motion are identical.

Appendix A
EQUATIONS OF MOTION AND CONTROL

The motion of a torpedo in yaw (Ref. 2 or 3) is given by

$$J_z\ddot{\psi} - N_r\dot{\psi} - N_\beta\beta = N_{\delta_r}\delta_r$$

and

$$(Vm_L - Y_r)\dot{\psi} - Vm_T\dot{\beta} - Y_\beta\beta = Y_{\delta_r}\delta_r$$

These are often referred to as the simplified motion equations.

Specifically it is assumed that (1) the angle of attack in pitch and the roll angle are small so that $r = \dot{\psi}$, as in Ref. 2; (2) the force Y, perpendicular to the torpedo axis and in the yaw plane, is given by $Y = Y_\beta\beta + Y_r r + Y_{\delta_r}\delta_r$ and the moment N about the torpedo center of gravity is $N = N_\beta\beta + N_r r + N_{\delta_r}\delta_r$; (3) the torpedo speed and mass are constant; (4) the angle of attack in yaw (β) is small so that $\sin\beta \simeq \beta$ and $\cos\beta \simeq 1$. Eliminating β and substituting the dimensionless coefficients (Ref. 2) leads to the following equation for the yaw angle ψ:

$$\dddot{\psi} + \frac{V}{\ell}\left(\frac{Y'_\beta}{m'_T} - \frac{N'_r}{J'_z}\right)\ddot{\psi} + \frac{V^2}{\ell^2}\left(\frac{Y'_r N'_\beta - m'_L N'_\beta - Y'_\beta N'_r}{m'_T J'_z}\right)\dot{\psi} \tag{17}$$

$$= \frac{V^2}{\ell^2}\,\frac{N'_{\delta_r}}{J'_z}\,\dot{\delta}_r + \frac{V^3}{\ell^3}\left(\frac{Y'_\beta N'_{\delta_r} - Y'_{\delta_r} N'_\beta}{m'_T J'_z}\right)\delta_r$$

It may be assumed without any loss of generality that the desired heading is $\psi = 0$, so that Eq. 17 is the equation for the error in heading. It is then necessary to control ψ to zero. Assuming the control

(18) $$\delta_r = \frac{K}{\frac{x_\delta}{\ell} Y'_{\delta_r}} \psi$$

and referring to the equations

(19) $$Y'_\beta = Y'_{\beta_B} + L'_{\beta_T}$$

$$Y'_r = Y'_{r_B} - \frac{x_\rho}{\ell} L'_{\beta_T}$$

$$N'_{\delta_r} = \frac{x_\delta}{\ell} Y'_{\delta_r}$$

$$N'_\beta = N'_{\beta_B} + \frac{x_\rho}{\ell} L'_{\beta_T}$$

$$N'_r = N'_{r_B} - \left(\frac{x_\rho}{\ell}\right)^2 L'_{\beta_T}$$

Eq. 17 becomes

(20) $$\dddot{\psi} + (a + bL'_{\beta_T})\ddot{\psi} + (c + dL'_{\beta_T} + eK)\dot{\psi} + K(f + gL'_{\beta_T})\psi = 0$$

where

$$a = \frac{V}{\ell}\left(\frac{Y'_{\beta_B}}{m'_T} - \frac{N'_{r_B}}{J'_z}\right)$$

$$b = \frac{V}{\ell}\left[\frac{1}{m'_T} + \frac{(x_\rho/\ell)^2}{J'_z}\right]$$

$$c = \frac{V^2}{\ell^2} \frac{1}{m'_T J'_z}(Y'_{r_B} N'_{\beta_B} - m'_L N'_{\beta_B} - Y'_{\beta_B} N'_{r_B})$$

$$d = \frac{V^2}{\ell^2} \frac{x_\rho}{\ell} \frac{1}{m'_T J'_z}\left(Y'_{r_B} - N'_{\beta_B} - m'_L + \frac{x_\rho}{\ell} Y'_{\beta_B} - \frac{\ell}{x_\rho} N'_{r_B}\right)$$

$$e = -\frac{V^2}{\ell^2}\frac{1}{J'_z}$$

$$f = -\frac{V^3}{\ell^3}\frac{1}{m'_T J'_z}\left(Y'_{\beta_B} - \frac{\ell}{x_\delta} N'_{\beta_B}\right)$$

$$g = -\frac{V^3}{\ell^3}\frac{1}{m'_T J'_z}\left(1 - \frac{x_\rho}{x_\delta}\right)$$

Now $x_\delta \simeq 1.05 x_\rho$ and $L'_{\beta_T} \approx 2$ so that g is negligible in comparison with f. Equation 20 is then reduced to

$$\dddot{\psi} + (a + bL'_{\beta_T})\ddot{\psi} + (c + dL'_{B_T} + eK)\dot{\psi} + (Kf)\psi = 0 \tag{21}$$

Appendix B

DEVELOPMENT OF STABILITY CRITERIA

The necessary and sufficient condition for stability of the solution $\psi = 0$ of Eq. 21, Appendix A, is that the real part of the roots of $m^3 + (a + bL'_{\beta_T})m^2 + (c + dL'_{\beta_T} + eK)m + Kf = 0$ be negative. To verify this, let the roots be m_1, m_2, and m_3 and consider the following cases:

1. If m_1, m_2, and m_3 are all real and distinct, then

$$\psi = Ae^{m_1 t} + Be^{m_2 t} + Ce^{m_3 t}$$

2. If $m_2 = m_3$ and $m_1 \neq m_2$, then

$$\psi = Ae^{m_1 t} + (B + Ct)e^{m_2 t}$$

3. If $m_1 = m_2 = m_3$, then

$$\psi = (A + Bt + Ct^2)e^{m_1 t}$$

4. If m_2 and m_3 are conjugate imaginaries $\gamma \pm \lambda i$, then

$$\psi = Ae^{m_1 t} + (B \cos \lambda t + C \sin \lambda t)e^{\gamma t}$$

In each of these cases the exponent must be negative in order that $\psi \rightarrow 0$ as $t \rightarrow \infty$, and conversely, which is simply the statement that the real part of the roots is negative. By noting the relationship between the roots and the coefficients, it is easy to derive equivalent necessary and sufficient conditions in terms of the coefficients, i.e.,

1. $a + bL'_{\beta_T} > 0$

2. $c + dL'_{\beta_T} + eK > 0$

3. $Kf > 0$

4. $(a + bL'_{\beta_T})(c + dL'_{\beta_T} + eK) > Kf$

For torpedo configurations with tail controls, a, b, and d are positive and c, e, and f are negative. Hence condition 1 is superfluous, condition 3 requires a negative K, and conditions 2 and 4 will be satisfied for a sufficiently large L'_{β_T}.

Let the solution of Eq. 21, Appendix A, be

$$\psi = A \exp(m_1 t) + \left[B \cos\left(\omega\sqrt{1 - \zeta^2}\, t\right) + C \sin\left(\omega\sqrt{1 - \zeta^2}\, t\right)\right] \exp(-\omega\zeta t) \tag{22}$$

where m_1 and $-\omega\zeta \pm \omega\sqrt{1 - \zeta^2}\, i$ are the roots of the auxiliary equation

$$m^3 + (a + bL'_{\beta_T})m^2 + (c + dL'_{\beta_T} + eK)m + Kf = 0 \tag{23}$$

The natural frequency is ω and ζ the damping factor. The root m_1 must be negative for stability. The relationships between the roots and the coefficients give

$$Kf = -m_1\omega^2 \tag{24}$$

$$c + dL'_{\beta_T} + eK = \omega^2 - 2m_1\omega\zeta$$

and

$$a + bL'_{\beta_T} = -m_1 + 2\omega\zeta$$

The solution of this set of equations is

$$L'_{\beta_T} = \frac{2e\zeta\omega^3 + (f - 4f\zeta^2 - ae)\omega^2 + 2af\zeta\omega - cf}{be\omega^2 - 2bf\zeta\omega + df} \tag{25}$$

(26) $$K = \frac{b\omega^4 - 2d\zeta\omega^3 + (da - bc)\omega^2}{be\omega^2 - 2bf\zeta\omega + df}$$

and

(27) $$m_1 = \frac{-fK}{\omega^2}$$

The author of Motion Equations for Torpedoes (Ref. 3) suggested that a response generally suitable for well-controlled torpedoes is given when $\zeta = \cos 45° = 0.707$ and $\omega = 2\pi$. For these values let the solutions of Eq. 24 be $L'_{\beta_T} = L^*$, $K = K^*$ and $m_1 = m^*_1$. Then from Eq. 25, 26, and 27

(28) $$L^* = \frac{f(8.886a - c - 39.48) + e(350.7 - 39.48a)}{f(d - 8.886b) + 39.48be}$$

(29) $$K^* = \frac{b(1558 - 39.48c) + d(39.48a - 350.7)}{f(d - 8.886b) + 39.48be}$$

and

(30) $$m^*_1 = \frac{-fK^*}{39.48}$$

The solution of the yaw-error equation for these values of L'_{β_T}, K, and m_1 is

(31) $$\psi = A \exp(m^*_1 t) + (B \cos 4.443t + C \sin 4.443t) \exp(-4.443t)$$

where A, B, and C are constants determined by the initial conditions. The term $A \exp(m^*_1 t)$ will rapidly approach zero if $-m^*_1$ is sufficiently large. For instance, it will be reduced to one tenth of its initial value in 0.2 second if $-m^*_1 = 11.5$. Hence, if $-m^*_1$ is approximately of this magnitude or larger, its effect is negligible after a very short period of time, and the torpedo will be comparable to those considered in this report.

Thus a torpedo, designed to the criterion $L'_{\beta_T} = L^*$ and having the control $\delta_r = [K^*/(x_\delta/\ell)Y'_{\delta_r}]\psi$, will control a yaw error to zero according to Eq. 31, provided the rudder is free to deflect to the angle prescribed by the control equation. However, the rudder is often limited to a certain maximum deflection, say $\pm\delta_{r_o}$ (the "±" indicates equal rudder-deflection limits in both directions), in which case the torpedo will respond according to Eq. 31 if and only if

$$(32) \qquad \frac{-\frac{x_\delta}{\ell} Y'_{\delta_r}\delta_{r_o}}{K^*} \leq \psi \leq \frac{\frac{x_\delta}{\ell} Y'_{\delta_r}\delta_{r_o}}{K^*}$$

This may be referred to, quite descriptively, as the "band of proportional control." When the angular error falls outside this band, the equation of motion becomes, for $L'_{\beta_T} = L^*$ (and assuming the error is positive, so that the rudder is positioned at $+\delta_{r_o}$)

$$(33) \qquad \dddot{\psi} + (a + bL^*)\ddot{\psi} + (c + dL^*)\dot{\psi} = -f\frac{x_\delta}{\ell} Y'_{\delta_r}\delta_{r_o}$$

The necessary and sufficient condition for stability of the steady-state solution of Eq. 33

$$(34) \qquad \dot{\psi}_{ss} = \frac{-f\frac{x_\delta}{\ell} Y'_{\delta_r}\delta_{r_o}}{c + dL^*}$$

is that $c + dL^*$ is positive. For each of the bodies considered in this report L^* was sufficiently large to satisfy this condition and therefore this restriction is not expected to affect the use of the criteria.

Often the steady-state turn rate per degree of rudder deflection ($\dot{\psi}_{ss}/\delta_r$) is a design specification. When $\delta_r = \delta_{r_o}$, from Eq. 34, Y'_{δ_r} is determined as

$$Y'_{\delta_r} = \frac{\dot{\psi}_{ss}}{\delta_{r_o}} \left(\frac{c + dL^*}{-f \frac{x_\delta}{\ell}} \right) \tag{35}$$

and this in turn will determine the band of proportional control from Expression 32.

Appendix C

SAMPLE COMPUTATION OF CRITERIA

According to Ref. 1 and 2 and the report of footnote 2, and the assumptions of the section "Parameters of Sample Torpedoes," Body 1.1 has the following dimensionless coefficients and physical characteristics:

$Y'_{\beta_B} = 0.54$, $Y'_{r_B} = 0.30$, $N'_{\beta_B} = 1.10$, $N'_{r_B} = -0.15$

$x_\rho/\ell = -0.446$, $x_\delta/\ell = -0.468$, $V/\ell = 8.906/\text{sec}$, $J'_z = 0.207$

$m'_L = 2.006$, $m'_T = 3.454$, $d = 1.012$ ft

$\ell = 7.590$ ft, $\ell_n = 0.304$ ft, $\ell_c = 4.554$ ft, $\ell_t = 2.732$ ft

Therefore

$$\eta = \frac{\ell_n}{\ell_n + \ell_t} = \frac{0.304}{0.304 + 2.732} = 0.1$$

$$\frac{\ell_p}{d} = \frac{\ell_n + \ell_t}{d} = \frac{0.304 + 2.732}{1.012} = 3$$

and

$$\frac{\ell_c}{\ell} = \frac{4.554}{7.590} = 0.6$$

The quantities a, b, c, d, e, and f necessary to determine L^*, K^*, and $Y'_{\delta_r}/(\dot{\psi}_{ss}/\delta_{r_o})$ are given by Appendix A as:

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$$a = 8.906\left(\frac{0.54}{3.454} + \frac{0.15}{0.207}\right) = 7.845$$

$$b = 8.906\left[\frac{1}{3.454} + \frac{(0.446)^2}{0.207}\right] = 11.136$$

$$c = \frac{(8.906)^2}{3.454(0.207)}[0.30(1.1) - 2.006(1.1) + 0.54(0.15)] = -199.24$$

$$d = \frac{(8.906)^2(-0.446)}{3.454(0.207)}\left[0.30 - 1.10 - 2.006 - (0.446)(0.54) - \frac{0.15}{0.446}\right] = 167.38$$

$$e = -\frac{(8.906)^2}{0.207} = -383.17$$

$$f = \frac{-(8.906)^3}{3.454(0.207)}\left(0.54 + \frac{1.1}{0.468}\right) = -2855.3$$

Substituting in Eq. 7

$$L^* = \frac{-2855.3[8.886(7.845) + 199.24 - 39.48] - 383.17[350.7 - 39.48(7.845)]}{-2855.3[167.38 - 8.886(11.136)] + 39.48(11.136)(-383.17)} = 1.84$$

Equation 8 gives

$$K^* = \frac{11.136[1558 + 39.48(199.24)] + 167.38[39.48(7.845) - 350.7]}{-2855.3[167.38 - 8.886(11.136)] + 39.48(11.136)(-383.17)} = -0.270$$

From Eq. 13

$$\frac{Y'_{\delta_r}}{\dot{\psi}_{ss}/\delta_{r_o}} = \frac{-199.24 + 167.38(1.84)}{-(2855.3)(0.468)} = -0.0819$$

If the approximate formulas are used, and, since $\eta = 0.1$, $\ell_p/d = 3$, and $\ell_c/\ell = 0.6$, Eq. 14, 15, and 16 give

$$L^* \simeq [0.909 - 0.37(0.1)](0.6) + 0.1 + 1.205 = 1.83$$

$$-K^* \simeq 0.019(3) - 0.016 + [0.1(3) + 0.082](0.6) = 0.270$$

and

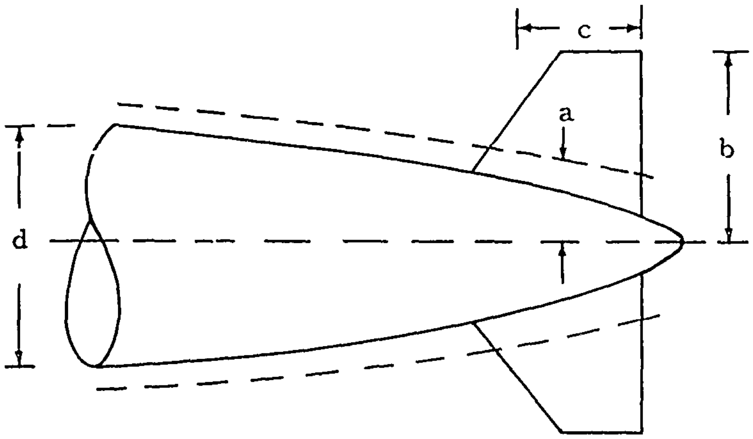
$$\frac{-Y'_{\delta_r}}{\dot{\psi}_{ss}/\delta_{r_o}} \simeq 0.007(3) + [0.0185(3) + 0.0448](0.6) = 0.0812$$

Appendix D

DIMENSIONLESS TAIL-LIFT COEFFICIENT AS A FUNCTION OF FIN SIZE

The quantity L'_{β_T} is defined as tail lift/ $(1/2)\rho AV^2$ and is referred to as the dimensionless tail-lift coefficient. Tail lift may be generated in many ways (by shroud rings, propellers, pumpjets, etc.), but the most frequent means is cruciform tail fins.

The following drawing defines the necessary dimensions for relating fin size to tail lift:



$$AR = \frac{2b}{c}$$

a = average body radius plus the average momentum thickness of the boundary layer (at the fins)

b = the half-span

c = average chord

d = maximum diameter

From pages 17 and 46 of Ref. 2

$$L'_{\beta_T} = \frac{2bc\ C_{L_\beta}}{A}\left(1 - \frac{a^2}{b^2}\right) = \frac{8bc\ C_{L_\beta}}{\pi d^2}\left(1 - \frac{a^2}{b^2}\right)$$

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where

$$C_{L_\beta} = \frac{2\pi b/c}{\sqrt{\frac{b^2}{c^2} + 1} + 1}$$

Therefore, upon substitution

$$L'_{\beta_T} = \frac{16 \frac{b^2}{d^2}\left(1 - \frac{a^2}{b^2}\right)}{\sqrt{\frac{b^2}{c^2} + 1} + 1}$$

This relationship is approximate and means of improving it are being studied, but it is accepted as reasonably accurate for preliminary-design purposes.

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